R.1 Equations

The first thing we want to do is review basic solving of equations. We will start with linear equations.

A few key ideas to remember is when solving equations we are usually trying to get the equation to look like **variable = number**. We do this by adding, subtracting, multiplying or dividing quantities on both sides of the equation. Also, we should remember that removing parenthesis and combining like terms are always good ideas to do first.

Example 1:

Solve.

a. $5-6x=-13$ b. $6y-1=2y+2$ c. $9n-3(2n-1)=15$

d. $5[2-(2x-4)]=2(5-3x)$ e. $5+3[1+2(2x-3)]=6(x+5)$

Solution:

a. We start by isolating the variable term by subtracting 5 from both sides, then we divide by –6 to get the **variable = number**.

$$\begin{aligned} 5-6x &= -13 \\ -5 \qquad & \quad -5 \\ \frac{-6x}{-6} &= \frac{-18}{-6} \\ x &= 3 \end{aligned}$$

We generally write solutions to equations in a solution set. So the solution is $\{3\}$.

b. This time we will need to move all the terms containing variables to the same side first. Then we can simply solve as we did in part a.

$$\begin{aligned} 6y-1 &= 2y+2 \\ -2y \qquad & \quad -2y \\ 4y-1 &= 2 \\ +1 & \quad +1 \\ \frac{4y}{4} &= \frac{3}{4} \\ y &= \tfrac{3}{4} \end{aligned}$$

So the solution set is $\left\{\tfrac{3}{4}\right\}$.

c. This time we start by distributing the –3 to remove the parenthesis. Then we combine like terms and continue as usual.

$$\begin{aligned} 9n-3(2n-1) &= 15 \\ 9n-6n+3 &= 15 \\ 3n+3 &= 15 && \text{Subtract 3 from both sides} \\ 3n &= 12 && \text{Divide by 3 on both sides} \\ n &= 4 \end{aligned}$$

So the solution set is $\{4\}$.

d. This example is a little more complicated. We need to use the distributive property several times to remove all the parenthesis. Then we can continue as we did before.

$$
\begin{aligned}
5[2-(2x-4)] &= 2(5-3x) \\
5[2-2x+4] &= 10-6x \\
5[-2x+6] &= 10-6x && \text{Combining like terms} \\
-10x+30 &= 10-6x \\
-4x &= -20 && \text{Add 6x and subtract 30 from both sides} \\
x &= 5 && \text{Divide by -4 on both sides}
\end{aligned}
$$

So the solution set is $\{5\}$.

e. Again we carefully simplify each side and then solve for the variable. We get

$$
\begin{aligned}
5+3[1+2(2x-3)] &= 6(x+5) \\
5+3[1+4x-6] &= 6x+30 \\
5+3[4x-5] &= 6x+30 && \text{Combining like terms} \\
5+12x-15 &= 6x+30 && \text{Combining like terms again} \\
12x-10 &= 6x+30 && \text{Subtract 6x and add 10 from both sides} \\
6x &= 40 && \text{Divide by 6 on both sides} \\
x &= \tfrac{40}{6} \\
x &= \tfrac{20}{3}
\end{aligned}
$$

So the solution set is $\left\{\frac{20}{3}\right\}$.

Another thing we want to remember is that when dealing with an equation that contains fractions, we usually want to start by clearing the fractions. We do this by multiplying both sides of the equation by the LCD (least common denominator).

<u>Example 2</u>:

Solve.

a. $2n-\frac{3}{4}=\frac{13}{4}$

b. $\dfrac{2x-13}{6}-x=\dfrac{x-2}{3}$

Solution:

a. Since the LCD is clearly 4, we multiply both sides by 4 and continue solving as in the last example. We get

$$
\begin{aligned}
4\cdot\left(2n-\tfrac{3}{4}\right) &= 4\cdot\left(\tfrac{13}{4}\right) \\
8n-3 &= 13 \\
8n &= 16 \\
n &= 2
\end{aligned}
$$

So the solution set is $\{2\}$.

b. This time the LCD is 6. We must be very careful when multiplying both sides by the LCD this time. The key is to multiply each term by the LCD and then carefully reduce. We proceed as follows

$$6\cdot\left(\frac{2x-13}{6}-x\right)=6\cdot\left(\frac{x-2}{3}\right)$$

$$6\cdot\left(\frac{2x-13}{6}\right)-6x=6\cdot\left(\frac{x-2}{3}\right)$$

$$2x-13-6x=2(x-2)$$

$$-4x-13=2x-4$$

$$-6x=9$$

$$x=-\tfrac{9}{6}$$

$$x=-\tfrac{3}{2}$$

So the solution set is $\left\{-\tfrac{3}{2}\right\}$.

Lastly, we want to solve applications by using linear equations.

Example 3:

Your mechanic charges you $278 for performing a 30,000-mile checkup on you car: This charge includes $152 for parts and $42 per hour for labor. Set up an equation and use it to find the number of hours the mechanic worked on your car.

Solution:
First we need to assign a variable to the quantity we are looking for. Lets let $n=$ number of hours worked. Now we can construct an equation. Since it is $152 for parts and $42 per hour labor we get the total cost is $42n+152$. Since the charge for the checkup is $278 we get the equation $42n+152=278$. Now we simply solve for *n*.

$$42n+152=278$$

$$42n=126$$

$$n=3$$

So the mechanic worked 3 hours on our car.

There are other, more complicated word problems. However, we will reserve solving those in the text when we review systems of equations, since it is easier to solve the more complex type by using a system of equations.

R.1 Exercises

Solve.

1. $3x-7=-19$
2. $12-5x=7$
3. $6x+20=0$
4. $5x-5=-5$
5. $4-x=4$
6. $3x-1-4x+7=-3$
7. $2x-20=7x-5$
8. $8-4x=x-9$
9. $9x-4(2x-3)=11$
10. $5(x+2)=4(x-1)$
11. $7-3(x-4)=2(4x+1)$
12. $3(2x-1)-(7x-8)=2x$
13. $2(x-5)+2-3(5-2x)=16$
14. $3(2x+3)=5-4(x-2)$

15. $5(1-x)-2(x+4)=3-2x$

16. $2(3-2x)=4-(3x-6)$

17. $6-(2x-1)=5(3x-4)$

18. $6(3-x)=5-(4x-7)$

19. $3(2x-1)-(4x+1)-2(5x-6)=8$

20. $3(2x-1)-(5x-4)-(7x+1)=-3$

21. $5(x-2)-2(4x-3)-(x+4)=-8$

22. $5(1-x)-2(x+4)-(7-2x)=x$

23. $4(3x-2)=2[3x-(2x-1)]$

24. $4[1-3(x+1)]=5(x+2)$

25. $3[2-5(x+1)]=-4(2x+1)$

26. $2[4-(3x+1)]=7(2x-1)$

27. $\dfrac{2}{3}x-1=-5$

28. $4-\dfrac{3}{4}x=-2$

29. $3x-\frac{1}{4}=\frac{3}{8}$

30. $5x+\frac{3}{2}=\frac{11}{8}$

31. $\frac{1}{3}x+\frac{3}{4}=\frac{11}{8}$

32. $\frac{4}{5}x-\frac{3}{15}=\frac{11}{10}$

33. $\dfrac{3}{8}x+\dfrac{1}{2}=\dfrac{1}{4}x+2$

34. $\dfrac{5}{6}x-\dfrac{2}{3}=\dfrac{3}{4}x+2$

35. $\dfrac{1}{2}x+\dfrac{2}{5}=\dfrac{1}{4}x-\dfrac{2}{3}$

36. $\dfrac{7}{2}x+\dfrac{2}{7}=\dfrac{1}{3}x-2$

37. $\dfrac{t}{6}+\dfrac{t}{8}=1$

38. $\dfrac{t}{5}-\dfrac{t}{28}=1$

39. $\dfrac{t+4}{14}=\dfrac{2}{7}$

40. $\dfrac{x+2}{12}=\dfrac{5}{4}$

41. $\dfrac{5-4x}{3}=\dfrac{x+2}{4}$

42. $\dfrac{2-3x}{5}=\dfrac{5x-1}{6}$

43. $\dfrac{25-4x}{3}=\dfrac{5x+12}{4}+6$

44. $\dfrac{8-3x}{4}-4=\dfrac{x}{6}$

45. $\dfrac{x-1}{8}-\dfrac{x}{4}=\dfrac{x-2}{2}$

46. $\dfrac{x-5}{9}-\dfrac{x}{3}=\dfrac{x+4}{27}$

47. $\dfrac{2x-5}{7}-\dfrac{x+1}{14}=\dfrac{2x-4}{3}$

48. $\dfrac{2x+3}{18}-\dfrac{3x-1}{14}=\dfrac{7x-4}{3}$

49. 6 times a number is increased by 10. The result is 94. Find the number.

50. 4 less than three times a number is 32. Find the number.

51. 10 less than twice a number is 100. Find the number.

52. 3 times a number is increased by 1. The result is 19. Find the number.

53. A union charges monthly dues of $3 plus $.18 for each hour worked during the month. A union member’s dues for July were $31.80. Use an equation to find how many hours were worked in July.

54. The monthly income for a manager of an apartment complex was $3500. This includes the manager’s base salary of $2500 plus a 2.5% bonus on total sales. Set up an equation and use it to find the managers total sales for the month.

55. A technical hotline charges a customer $8 plus $.40 per minute to answer questions about software. Use an equation to find out how many minutes a customer was on with the hotline if they were charged $22.

56. Budget plumbers charged $465 for replacing a water heater and replacing pipes to the water heater. The charge included $365 for materials and $40 per hour for labor. Use an equation to find how many hours of labor were charged.

57. The monthly income for a manager of a mobile home dealership was $3500. This includes the managers base salary of $2500 plus a 1% commission on total sales. Use an equation to find the sales for the month.

58. Your mechanic charges you $278 for performing a 30,000-mile checkup on you car: This charge includes $152 for parts and $42 per hour for labor. Set up an equation and use it to find the number of hours the mechanic worked on your car.

Use the formula $V = V_0 + 32t$ to answer the following questions, where V is the final velocity, V_0 is the initial velocity, of a falling object, and t is the time for the object to fall.

59. Find the time required for a falling object to increase in velocity from 16 ft/sec to 128 ft/sec.

60. Find the time required for a falling object to increase in velocity from 2 ft/sec to 58 ft/sec.

Use the formula $C = \frac{5}{9}(F - 32)$, where C is the Celsius temperature and F is the Fahrenheit temperature, to answer the following questions.

61. Find the Fahrenheit temperature when the Celsius temperature is $-40°$.

62. Find the Fahrenheit temperature when the Celsius temperature is $0°$.

63. The length of a rectangle is 5 ft more than the width. If the perimeter is 30 ft, what are the dimensions of the rectangle?

64. The width of a rectangle is 8 m less than the length. If the perimeter is 24 m, what are the dimensions of the rectangle?

65. The length of a rectangle is three less than two times the width. If the perimeter is 18 feet, what are the dimensions of the rectangle?

66. The length of a rectangle is twice the width. If the perimeter is 300 feet, what are the dimensions of the rectangle?

67. The height of a triangle is 3 cm. If the area is 30 cm^2, what is the length of the base of the triangle?

68. The base of a triangle is 15 cm. If the area is 75 cm^2, what is the length of the base of the triangle?

69. One side of a triangle is 3 times the length of the first. The other side is 8 m less than twice the first. If the perimeter is 22 m, what are the lengths of the sides of the triangle?

70. One side of a triangle is 2 times the length of the first. The other side is 6 m less than the first. If the perimeter is 26 m, what are the lengths of the sides of the triangle?