9.8 Applications of Functions

In this section we want to apply what we have learned about functions to real world problems, a.k.a. word problems.

There are two primary types of application problems we would like to look at, problems with the function given to us and problems were we must find the function and then use it.

Let us consider the first type, that is, where the function is given to us and we need to interpret functions and function values.

<u>Example 1</u>:

The number of bacteria in a refrigerated food is given by $N(T) = 20T^2 - 20T + 120$, for $2 \le T \le 14$ and where *T* is the temperature of the food in Celsius. Find the following:

a. What is the bacterium count when the temperature is $12°$?
b. At what temperature will the bacteria count be 240?

Solution:

a. If the temperature is $12°$, then that means that $T = 12$. Since we are looking for the number of bacteria, we want $N(12)$. So, we get

$$N(12) = 20(12)^2 - 20(12) + 120$$
$$= 2760$$

So, there are 2760 bacterium present when the temperature is $12°$ Celsius.

b. This time we want to know the temperature when we are given the bacteria count. Notice that the *N* values in the function represent the number of bacteria. This means that this time we are looking for *T* values that give us $N(T) = 240$. So we have the following

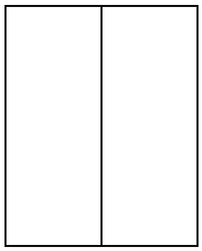
$$N(T) = 20T^2 - 20T + 120 = 240$$
$$20T^2 - 20T - 120 = 0$$
$$20(T^2 - T - 6) = 0$$
$$20(T - 3)(T + 2) = 0$$

So, $T = 3$ and $T = -2$. But, $2 \le T \le 14$. Therefore, we can eliminate –2 and we get that $T = 3$. Thus, there will be 2760 bacteria present when the temperature is $3°$ Celsius.

The other type of word problem we will separate into two different types.
The first type is a problem in which we must use previous information to construct a function and then use that function to answer some questions.

Example 2:

Two rectangular corral's are to be made from 100 yds of fencing as seen below.

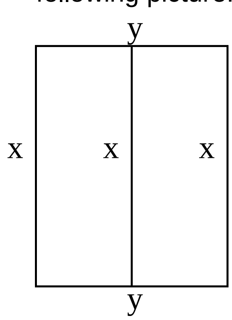


Find the following:

a. Determine a function for the total area as a function of the longer side.
b. Use part a. to determine the total area when the length of the longer side is 30 yds.
c. If the rancher wants the total area to be 400 sq. yds., what dimensions should be used to make the corral's?

Solution:

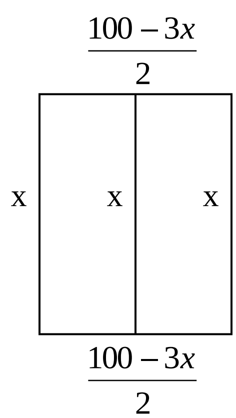
a. The first thing we should do is decide on some labels and variables. Lets call the longer side x, since that is what we want a function of, and the shorter side y. This gives the following picture:



The first thing we must do is express y in terms of x, that way we will only have to deal with one variable. So the total amount of fencing to be used can be represented by x+x+x+y+y. But we know the total amount to be used is 100yds.

Therefore, 100=3x+2y. So $y = \frac{100-3x}{2}$

Thus we have the picture



So, the total area of the rectangle is

$$A = l \cdot w$$

$$A(x) = x \cdot \frac{100-3x}{2}$$

Notice we have a function of the longer side, x.

b. Since x is the longer side, we are looking for $A(30)$. We get

$$A(30) = 30 \cdot \frac{100 - 3 \cdot 30}{2}$$
$$= 150$$

So, the area is 150 sq. yds.

c. Lastly, we have been given the area of 400 sq. yds. And this time we want the size of the sides. So we have $A(x) = 400$ and we need to find x. We get

$$A(x) = x \cdot \frac{100 - 3x}{2} = 400$$
$$x(100 - 3x) = 800$$
$$100x - 3x^2 = 800$$
$$3x^2 - 100x + 800 = 0$$
$$(3x - 40)(x - 20) = 0$$
$$x = \frac{40}{3} \qquad x = 20$$

So we have a choice of values. However, if $x = \frac{40}{3}$ then $y = 30$. But this can't be the case since we defined x to be the longer of the two sides. So thus $x = 20$ and therefore $y = 20$. So our answer is 20 yds X 20 yds.

The last type of word problem we will look at is called a <u>variation</u> problem.

There are four basic types of variation we would like to consider: Direct variation, Direct variation as an *n*th power, Inverse variation, and Joint variation. We can also combine several of these together to get Combined variation.

Variation

<u>Direct Variation</u> *y* varies directly as (or is proportional to) *x* means $y = kx$ for some *k*	<u>Inverse Variation</u> *y* varies inversely as (or is inversely proportional to) *x* means $y = \frac{k}{x}$ for some *k*
<u>Direct Variation as an *n*th power</u> *y* varies as the nth power of *x* (or *y* is proportional to the nth power of *x*) means $y = kx^n$ for some *k*	<u>Joint Variation</u> *z* varies jointly as (or is jointly proportional to) *x* and *y* means $z = kxy$ for some *k*

The constant *k* in a variation problem is called the <u>constant of proportionality</u>. We generally need to determine this constant first. We do this by using a given set of values for *x*, *y* and/or *z*.

<u>Example 3</u>:

The distance a ball rolls down a hill is directly proportional to the square of the time that it rolls. During the first second, a ball rolls down a hill a distance of 10 feet. How far will the ball roll during the first 3 seconds?

Solution:

Lets call the distance *d* and the time *t*. Then, by the above box we have the equation

$$d = kt^2$$

The first thing we need to do is to determine the constant *k.* We do this by using the given information in the problem. We see that when $t = 1$, $d = 10$. If we plug these into the above equation we get

$$10 = k(1)^2$$
$$k = 10$$

So our function for this situation is $d(t) = 10t^2$. We now are ready to answer the question of what is the distance when $t = 3$ seconds.

$$d(3) = 10(3)^2$$
$$= 90$$

Therefore the ball rolls 90 feet in the first 3 seconds.

Example 4:

In physics, the volume V of a gas varies directly as the temperature T and inversely as the pressure P. A certain gas has a volume of 275 cm^3 when the pressure is 20 g/cm^2 and temperature is 25^o. Find the temperature at which the same gas has a volume of 220 cm^3 and the pressure is 50 g/cm^2.

Solution:

First we need to generate the formula. Notice that we have three different variables. First of all, we have volume V of a gas varies directly as the temperature T. That gives us

$$V = k_1 T$$

Also, we have volume V of a gas varies inversely as the pressure P. That gives us

$$V = \frac{k_2}{P}$$

For simplicity we can combine the two formulas together and combine the constants to get the formula

$$V = \frac{kT}{P}$$

Now we must determine the constant *k*. We use the given information to do so, that is, V=275, P=20 and T=25

$$275 = \frac{k \cdot 25}{20}$$
$$5500 = k \cdot 25$$
$$220 = k$$

So the completed model for this gas is $V = \frac{220T}{P}$.

Now we can solve the problem. We wanted to know what the temperature is when V=220 and P=50. We substitute these values into the formula to get

$$220 = \frac{220T}{50}$$
$$11000 = 220T$$
$$T = 50$$

So the temperature would be 50º.

9.8 Exercises

1. The discount price *d* of a shirt is a function of the original price *p* where $d(p) = 0.75p$. What is the discount price of a shirt that has an original price of $40? What is the original price if the discount price is $37.50?

2. The cost of producing *x* number of footballs is given by $C(x) = 25 + 2x$, where *C* is in dollars. What is the cost of producing 100 footballs? How many footballs can be produced if we only have $200 to spend on cost?

3. The cost of producing *x* number of laptop computers is given by $C(x) = 250 + 150x$, where *C* is in dollars. What is the cost of producing 20 laptop computers? How many can be produced if we only have $1200 to spend on cost?

4. The number of sales per year, in millions, of DVD's after 2000 is given by $N(t) = 45t - 12$ where $t = 0$ corresponds to the year 2000. How many DVD's were sold in the year 2002? In what year will the sales reach 475 million DVD's?

5. The number of board feet in a 16 foot long tree is approximated by the model $F(d) = 0.77d^2 - 1.32d - 9.31$ where *F* is the number of feet and *d* is the diameter of the log. How many board feet are in a log with diameter 12 inches? 16 inches?

6. The number of horsepower needed to overcome a wind drag on a certain automobile is given by $N(s) = 0.005s^2 + 0.007s - 0.031$, where *s* is the speed of the car in miles per hour. How much horsepower is needed to overcome the wind drag on this car if it is traveling 50 miles per hour? 100 miles per hour?

7. The number of baseball games that must be scheduled in a league with *n* teams is given by $G(n) = \frac{n^2 - n}{2}$ where each team plays every other team exactly once. A league schedules 15 games. How many teams are in the league? How many games must be scheduled if there are a total of 10 teams?

8. The height in feet of a bottle rocket is given by $h(t) = 160t - 16t^2$ where *t* is the time in seconds. How long will it take for the rocket to return to the ground? What is the height after 2 seconds?

9. A foul ball leave the end of a baseball bat and travels according to the formula $h(t) = 64t - 16t^2$ where *h* is the height of the ball in feet and *t* is the time in seconds. How long will it take for the ball to reach a height of 64 feet in the air?

10. The distance you can see to the horizon is a function of the altitude at which you are positioned. This function is $d(h)=1.22\sqrt{h}$ where *d* is the distance in miles and *h* is the altitude in feet. How far can you see if you are at an altitude of 10,000 feet? At what altitude will you be able to see 100 miles?

11. The equation for the distance that can be seen from a periscope can see is $d(h)=1.22\sqrt{h}$ where *d* is the distance in miles and *h* is the height above the water in feet. How far above the water would a periscope have to be to see an enemy ship 3.8 miles away? What height would the captain want the periscope if he wanted to see more than 10 miles?

12. The period of a pendulum is the time it takes for the object to make one full swing, i.e. starting from the left, swinging to the right, then finishing on the left again, or vice versa. The formula for the period of a pendulum is $T(l)=2\pi\sqrt{\frac{l}{32}}$ where *T* is the period in seconds and *l* is the length of the pendulum in feet. What is the period for a pendulum in which the length is 2 feet? What length will produce a period of 4 seconds?

13. The period of a pendulum is the time it takes for the object to make one full swing, i.e. starting from the left, swinging to the right, then finishing on the left again, or vice versa. The formula for the period of a pendulum is $T(l)=2\pi\sqrt{\frac{l}{32}}$ where *T* is the period in seconds and *l* is the length of the pendulum in feet. What is the period for a pendulum in which the length is 1 foot? What length will produce a period of 2 seconds?

14. A manufacturer of tennis balls has a daily cost of $C(x)=200-10x+0.01x^2$ where *C* is the total cost in dollars and *x* is the number of tennis balls produced. What is the cost of producing 1000 tennis balls? What is the cost per ball for producing 1000 balls?

15. The value of Jennifer's stock portfolio is given by the function $v(t)=50+73t-3t^2$, where *v* is the value of the portfolio in hundreds of dollars and *t* is the time in months. How much money did Jennifer start with? How much does she have after 10 months? Does Jennifer ever lose all of her money?

16. The value of Jon's stock portfolio is given by the function $v(t)=50+77t+3t^2$ where *v* is the value of the portfolio in hundreds of dollars and *t* is the time in months. How much money did Jon start with? How much does he have after 10 months? Does Jon ever lose all of his money?

17. The length of a rectangle is three more than twice the width.
    a. Determine a function for the total area as a function of the width.
    b. Use part a. to determine the total area when the width is 2 m.
    c. Use part a. to determine the dimensions that will give a total area of 27 m^2.

18. The width of a rectangle is five less than twice the length.
    a. Determine a function for the total area as a function of the length.
    b. Use part a. to determine the total area when the length is 3 ft.
    c. Use part a. to determine the dimensions that will give a total area of 88 ft^2.

19. The length of a rectangle is one more than the width.
    a. Determine a function for the total area as a function of the length.

b. Use part a. to determine the total area when the length is 10 ft.
c. Use part a. to determine the dimensions that will give a total area of 20 ft^2.

20. The base of a triangle is one more than four times the height.
    a. Determine a function for the total area as a function of the height.
    b. Use part a. to determine the total area when the height is 12 cm.
    c. Use part a. to determine the dimensions that will give a total area of 9 cm^2.

21. The height of a triangle is two more than three times the base.
    a. Determine a function for the total area as a function of the base.
    b. Use part a. to determine the total area when the base is 5 yds.
    c. Use part a. to determine the dimensions that will give a total area of 28 yds^2.

22. The base of a triangle is two less than five times the height.
    a. Determine a function for the total area as a function of the height.
    b. Use part a. to determine the total area when the height is 2 cm.
    c. Use part a. to determine the dimensions that will give a total area of 36 cm^2.

23. The volume *V* and surface area *S* of a sphere of radius *r* are given by $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$. Determine *V* as a function of *S.* Use this result to calculate the volume when the surface area is 16π.

24. The volume *V* and surface area *S* of a right circular cylinder of radius *r* and height h are given by $V = \pi r^2 h$ and $S = 2\pi r^2 + 2\pi rh$. If the volume of a right circular cylinder is 10 in^3, determine *S* as a function of *r*. Use this result to calculate the surface area when the radius is 2 inches.

25. The perimeter of a rectangle is 50 yds.
    a. Determine a function for the area in terms of the width.
    b. Use part a. to determine the total area when the width is 3 yds.
    c. Use part a. to determine the dimensions that will give a total area of 46 yds^2.

26. The perimeter of a rectangle is 70 m.
    a. Determine a function for the area in terms of the length.
    b. Use part a. to determine the total area when the length is 13 m.
    c. Use part a. to determine the dimensions that will give a total area of 300 m^2.

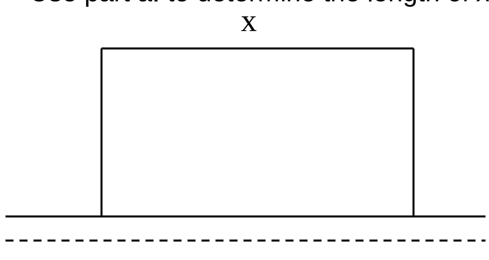
27. Determine a function for the area of a circle in terms of the circumference.
    a. Use the function to determine the total area when the circumference is 16 yds.
    b. Use the function to determine the circle that will give a total area of $1/16\pi$ yds^2.

28. A piece of wire is bent into the shape of a square.
    a. Determine a function for the area of the square in terms of the length of the wire.
    b. Use part a. to determine the total area when the length of the wire is 8 cm.
    c. Use part a. to determine the length of wire that will give an area of 16 cm^2.

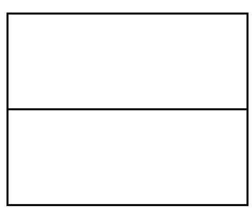
29. A piece of wire is bent into the shape of a circle.
    a. Determine a function for the area of the circle in terms of the length of the wire.
    b. Use part a. to determine the total area when the length of the wire is 4 inches.
    c. Use part a. to determine the length of wire that will give an area of $1/4\pi$ in^2.

30. Three hundred feet of fencing is available to enclose a rectangular yard along side of the St. John's River, which is one side of the rectangle as seen below.
    a. Determine the function for the area of the yard in terms *x*.
    b. Use part a. to determine the area when the side given is 50 feet.
    c. Use part a. to determine the length of *x* that will give an area of 10,000 ft^2.

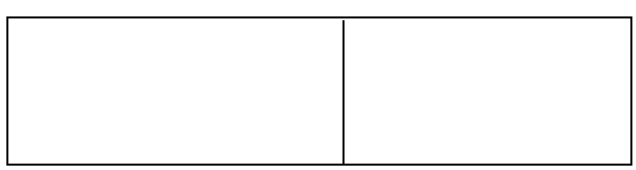
31. Five hundred feet of fencing is available to enclose a rectangular lot along side of highway 65. Cal Trans will supply the fencing for the side along the highway, so only three sides are needed as seen below.
    a. Determine the function for the area of the lot in terms *x*.
    b. Use part a. to determine the area when the side given is 250 feet.
    c. Use part a. to determine the length of *x* that will give an area of 40,000 ft^2.



32. Two rectangular pens are to be made from 200 yds of fencing as seen below.
    a. Determine a function for the total area as a function of the longer side.
    b. Use part a. to determine the total area when the length of the longer side is 60 yds.
    c. Use part a. to determine the length of the dimensions that will give an area of 2500 yds^2.

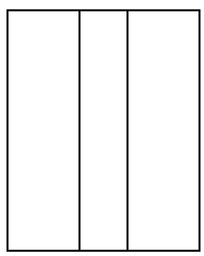


33. Two rectangular lots are to be made from 400 ft of fencing as seen below.
    a. Determine a function for the total area as a function of the shorter side.
    b. Use part a. to determine the total area when the length of the shorter side is 15 feet.
    c. Use part a. to determine the length of the dimensions that will give an area of 3400 ft^2.



34. Three rectangular corrals are to be made from 800 meters of fencing as seen below.
    a. Determine a function for the total area as a function of the longer side.
    b. Use part a. to determine the total area when the length of the longer side is 150 meters.
    c. Use part a. to determine the length of the dimensions that will give a total area of 8750 m^2.

35. Three rectangular corrals are to be made from 100 yards of fencing as seen below.
    a. Determine a function for the total area as a function of the longer side.
    b. Use part a. to determine the total area when the length of the longer side is 18 yards.
    c. Use part a. to determine the length of the dimensions that will give a total area of 200 yds^2.



Hooke's Law for springs states that the distance a spring is stretched (or compressed) varies directly as the force on the spring. Use this Law to answer the following questions.

36. A force of 10 lbs stretches a spring 5 inches. How far will a force of 30 lbs stretch the spring?

37. A force of 15 grams stretches a spring 5 cm. How far will a force of 9 grams stretch the spring?

38. A force of 50 lbs compresses a spring 1.5 inches. How much will the spring be compressed by a weight of 20 lbs?

39. A force of 9.5 lbs compresses a spring 2 inches. What force will compress the spring 5 inches?

40. A force of 14 grams stretches a spring 3 cm. What force will stretch the spring 8 cm?

41. The pressure on a diver in the water varies directly as the depth. If the pressure is 9.5 lb/in^2 when the depth is 15 feet, what is the pressure when the depth is 40 feet?

42. The income of a lawyer varies directly as the number of hours worked. If the lawyer earns $450 for working 5 hours, how much will the lawyer earn by working 12 hours?

43. The length of a rectangle of fixed area varies inversely as the width. If the length of a rectangle is 9 feet when the width is 5 feet, find the length when the width is 7 feet.

44. The speed of a gear varies inversely as the number of teeth. If a gear that has 35 teeth makes 40 revolutions per minute, how many revolutions per minute will a gear that has 40 teeth make?

45. The circumference of a circle is directly proportional to its diameter. A circle has a circumference of 6.59 inches when it has a diameter of 2.1 inches. What is the diameter when the circumference is 8.3 inches?

46. The temperature of a gas varies directly with its pressure. A temperature of 210^{o} produces a pressure of 15 lbs/in^2. What is the pressure if the temperature is 140^{o}?

47. A company found that the demand for its product is inversely proportional to the price. When the price is $3, the demand is 900 units. What is the demand when the price is $4?

48. The velocity of a free falling object varies directly as the time it has fallen. If the velocity of a falling object is 160 ft/sec after 5 seconds, what is the time required to have a velocity of 128 ft/sec?

49. The time for a car to travel between two cities is inversely proportional to the rate of travel. If it takes 8 hours to travel from San Francisco to Los Angeles at a rate of 60 mph, how long would it take traveling at 75 mph?

50. The distance an object falls varies directly as the square of the time of the fall. If an object falls 256 feet in 4 seconds, how long will it take to fall 64 feet?

51. The repulsive force between the north poles of two magnets is inversely proportional to the square of the distance between them. If the repulsive force is 40 lbs when the distance is 2 inches, what is the distance when the repulsive force is 10 lbs?

52. The intensity of a light source is inversely proportional to the square of the distance between them. If the intensity is 4 foot candles at a distance of 2 feet, what is the distance when the intensity is ½ foot candles?

53. The gravitational pull of an object on the earth is inversely proportional to its distance from the center of the earth. If an astronaut weights 165 lbs on earth, how much will he weigh when he is 500 miles above the surface of the earth? (Assume the radius of the earth is 4000 miles)

54. The pressure in a liquid varies jointly as the depth and the density of the liquid. If the pressure is 80 lb/in^2 when the depth is 20 inches and the density is 2.1, what is the pressure of the liquid when the depth is 30 inches and the density remains the same?

55. The current in a wire varies directly as the voltage and inversely as the resistance. If the current is 15 amps when the voltage is 165 volts and the resistance is 12 ohms, what is the current when the voltage is 110 volts and the resistance is 4 ohms?

56. The tension of a string being twirled with a weight on one end is directly proportional to the square of the speed of the weight and inversely proportional to the length of the string. If the tension is 24 lbs when the string is 2 feet and the speed is 4 ft/sec, what is the tension when the length is increased to 4 feet, and the speed is increased to 15 ft/sec?

57. The strength of a rectangular beam varies jointly as the square of the width and the square of its depth and inversely as its length. If the strength of a beam 2 inches wide,

12 inches deep and 12 feet long is 1200 lbs, what is the strength of a beam that is 5 inches wide, 6 inches deep and 10 feet long?

58. The current in a wire varies directly as the voltage and inversely as the resistance. If the current is 20 amps when the voltage is 210 volts and the resistance is 21 ohms, what is the voltage when the current is 25 amps and the resistance is 8 ohms?

59. The pressure in a liquid varies jointly as the depth and the density of the liquid. If the pressure is 50 g/cm^2 when the depth is 15 cm and the density is 1.4, what is the depth in the liquid when the pressure is 65 g/cm^2 and the density is doubled?

60. The strength of a rectangular beam varies jointly as the square of the width and the square of its depth and inversely as its length. If the strength of a beam 4 inches wide, 6 inches deep and 12 feet long is 960 lbs, what is the depth required of a beam that is 3 inches wide, 8 feet long to have a strength of 2250 lbs?

61. The tension of a string being twirled with a weight on one end is directly proportional to the square of the speed of the weight and inversely proportional to the length of the string. If the tension is 50 lbs when the string is 2 feet and the speed is 10 ft/sec, what is the speed when the length is decreased to 1 feet, if the tension 100 lbs?

62. Newtons law of gravitation states the following: The gravitation between two bodies varies jointly as the masses of the two bodies and inversely as the square of the distance between them. Write the formula for Newtons law of gravitation.