9.1 Linear Equations in Two Variables

The first thing we would like to do in this chapter is review some of the basic facts of linear equations in two variables.

First, we will recall how to graph a linear equation in two variables.

<u>Example 1</u>:

Graph 3x+2y=1.

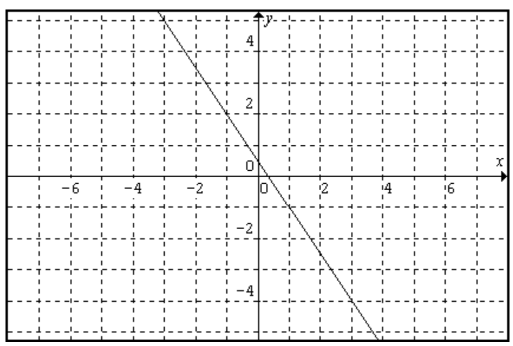
Solution:

The easiest way to graph an equation of any type is by plotting points. We can choose any values that we want for x and y. So we will choose –1, 0 and 1 for x. We summarize these values in a table of values.

x	y
-1	2
0	½
1	-1

$$3(-1)+2y=1 \qquad 3(0)+2y=1 \qquad 3(1)+2y=1$$
$$2y=4 \qquad 2y=1 \qquad 2y=-2$$
$$y=2 \qquad y=1/2 \qquad y=-1$$

Now we simply need to plot the points and connect them with a line.



Notice that the graph crosses both the x and y-axis. We call these values the intercepts of the graph.

Definition:
<u>x-intercepts</u>- The x-intercepts of a graph are the values at which the graph crosses the x-axis. To find the x-intercepts, Let y=0 in the equation, then solve for x.
<u>y-intercepts</u>- The y-intercepts of a graph are the values at which the graph crosses the y-axis. To find the y-intercepts, Let x=0 in the equation, then solve for y.

<u>Example 2</u>:

Graph x-2y=4 using the intercepts.

Solution:

First, we will find the x- and y-intercepts.

<u>x-intercept</u>: <u>y-intercept</u>:

We let y=0 and solve for x.

$$x - 2(0) = 4$$
$$x = 4$$

So the x-intercept it (4,0)

We let x=0 and solve for y.

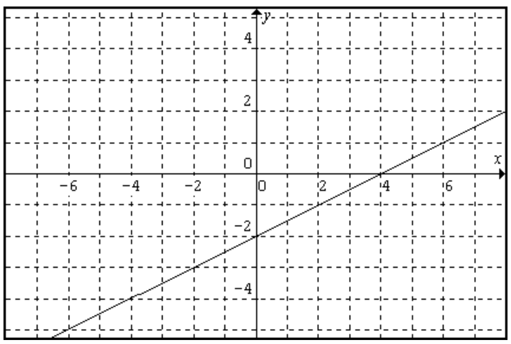
$$0 - 2y = 4$$
$$y = -2$$

So the y-intercept it (0,-2)

We will also find one more point just to verify that we have a line.
Lets use x=1. This gives us

$$1 - 2y = 4$$
$$-2y = 3$$
$$y = -1\tfrac{1}{2}$$

So we also have the point $\left(1, -1\tfrac{1}{2}\right)$. Now we simply plot and graph.

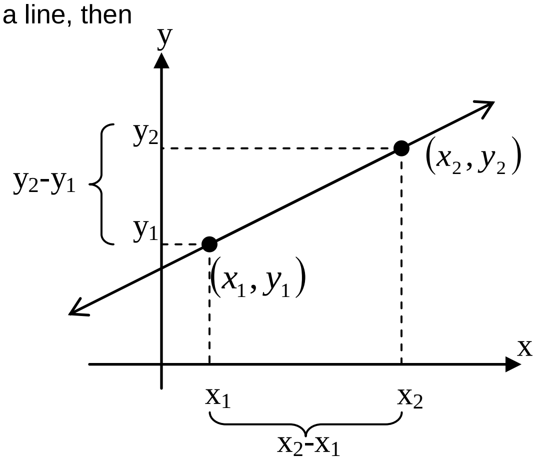


Notice that the lines in example 1 and example 2 are slanted in different directions. Also, notice that the line in example 1 is much steeper than the line in example 2. These ideas fall under something we call the slope of a line.

Definition: <u>Slope of a line</u>-

The slope of a line, m, is defined as follows:
Let (x_1, y_1) and (x_2, y_2) be two points on a line, then

$$m = \frac{change\ \ in\ \ the\ y\ direction}{change\ \ in\ \ the\ x\ direction} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$



We can see that this formula for slope would describe this idea of one line being steeper than another, since if a line changes very little in the y direction as compared with the x direction, it would be a more flat line and if it changed a lot in the y direction as compared with the x direction it would be a more steep line.

Example 3:

Find the slope of the line through the points given.

a. (0, 0), (-2, -1) b. (3, 6), (5, -2) c. (-4, 3), (4, 3) d. (1, 2), (1, 3)

Solution:

a. We first label the points (x_1, y_1) and (x_2, y_2). Lets let (0, 0) be (x_1, y_1), and (-2, -1) be (x_2, y_2). Now plugging into the formula for slope we get

$$\begin{aligned} m &= \frac{-1-0}{-2-0} \\ &= \frac{-1}{-2} \\ &= \frac{1}{2} \end{aligned}$$

So $m = \frac{1}{2}$.

b. Again we label the points (x_1, y_1) and (x_2, y_2). Let (3, 6) be (x_1, y_1), and (5, -2) be (x_2, y_2). This gives us

$$\begin{aligned} m &= \frac{-2-6}{5-3} \\ &= \frac{-8}{2} \\ &= -4 \end{aligned}$$

So $m = -4$.

c. Let (-4, 3) be (x_1, y_1), and (4, 3) be (x_2, y_2). We get

$$\begin{aligned} m &= \frac{3-3}{4-(-4)} \\ &= \frac{0}{8} \\ &= 0 \end{aligned}$$

So $m = 0$

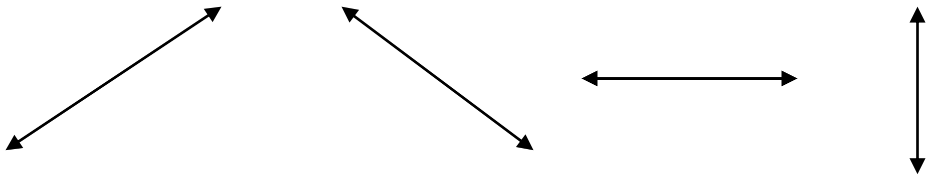
d. Let (1, 2) be (x_1, y_1), and (1, 3) be (x_2, y_2). We get

$$\begin{aligned} m &= \frac{3-2}{1-1} \\ &= \frac{1}{0} \end{aligned}$$

But we cannot have a zero on the denominator. This gives us an undefined slope.

We can see from the last example that there are four different types of slopes: positive, negative, zero, and undefined.

The geometric interpretation for this is as follows:



m positive   m negative   m zero   m undefined

Also, we can use the slope idea to graph lines. In order to do that we need the following

Slope-Intercept form of the equation of a line
The graph of $y = mx + b$ is a line whose slope is *m* and y-intercept is (0, *b*).

So, if we have the equation of a line, we simply need to solve it for y and then we can read the slope and the y-intercept right off. Then we can use these pieces of information to graph the line.

<u>Example 4</u>:

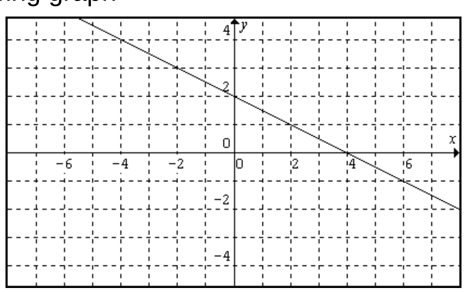
Graph $y = -\frac{1}{2}x + 2$ using the slope.

Solution:

Since the equation is already solved for y, it is already in slope intercept form. Thus we can read the slope and the y-intercept right off of it. We get

$m = -\frac{1}{2}$ y-intercept is (0, 2)

Remember that the slope is $m = \dfrac{change\ in\ y}{change\ in\ x}$. So it is easiest if we write $m = -\frac{1}{2} = \frac{-1}{2}$. This way we can account for the negative when graphing. Now we simply plot the y intercept and use the slope to graph other points. Change in y equaling –1 means go down one and change in x equaling 2 means go right 2. This gives the following graph



<u>Example 5</u>:

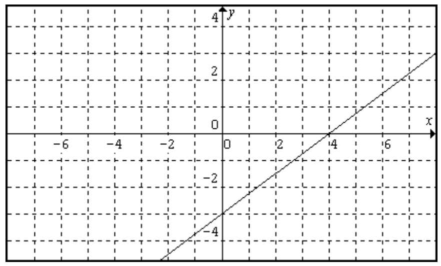
Graph $3x - 4y = 12$.

Solution:

Lets use the same method as the previous example. First though this equation needs to be put into slope intercept form. So we must first solve it for y.

$$3x - 4y = 12$$
$$-4y = -3x + 12$$
$$y = \tfrac{3}{4}x - 3$$

So we can see the slope is ¾ and the y intercept is –3. Graphing like before we start at the y intercept and go up 3 and right 4. We have



The last thing we would like to do is to generate the equation of a line based upon information that we could obtain from the graph. We first state two more forms of the equation of a line.

Point-Slope form of the equation of a line
If (x_1, y_1) is a point on the line with slope m, then $y - y_1 = m(x - x_1)$.

And

Standard Form of the equation of a line
If *a*, *b*, and *c* are integers and *a* is positive then the standard form of the equation of a line is $ax + by = c$.

Lets see some examples

<u>Example 6</u>:

Find the equation in slope intercept form for the line through (2, 3) and having slope –1/2.

Solution:

Since we have a point and the slope given to us, its clear we should use the point-slope formula to find the equation. We can then switch to slope intercept form by solving for y. We begin by labeling the point (2, 3) as (x_1, y_1) and slope m=-1/2. Plugging into the formula we get

$$y - 3 = -\tfrac{1}{2}(x - 2)$$
$$y - 3 = -\tfrac{1}{2}x + 1$$
$$y = -\tfrac{1}{2}x + 4$$

So the slope intercept form is $y = -\tfrac{1}{2}x + 4$.

<u>Example 7</u>:

Find the equation of the line in standard form that passes though (-2, -3) and (2, -12).

Solution:

The first thing that we notice is that two of the three forms of the equation of a line require the slope. So we should begin by finding the slope of the line though these two points. We label (-2, -3) as (x_1, y_1) and (2, -12) as (x_2, y_2). So we get

$$\begin{aligned} m &= \frac{-12-(-3)}{2-(-2)} \\ &= \frac{-9}{4} \end{aligned}$$

Now that we have the slope we can use this with either point to find the equation. We just put them into the point slope formula. We will use the point that is already labeled (x_1, y_1) for simplicity.

$$\begin{aligned} y-(-3) &= -\tfrac{9}{4}(x-(-2)) \\ y+3 &= -\tfrac{9}{4}x-\tfrac{9}{2} \end{aligned}$$

Since we want to get it into standard form we will clear the fractions (by multiplying by the LCD of 4) at this point and then move the variables to whatever side we need to in order to make the coefficient of x positive.

$$\begin{aligned} y+3 &= -\tfrac{9}{4}x-\tfrac{9}{2} \\ 4y+12 &= -9x-18 \\ 9x+4y &= -30 \end{aligned}$$

So the standard form of the equation of the line is $9x+4y=-30$.

9.1 Exercises

Find the x-and y-intercepts and use them to graph the equation.

1. $2x+y=4$
2. $x+3y=6$
3. $x-y=0$
4. $2x+3y=6$
5. $4x-y=2$
6. $5x-4y=-20$
7. $x+7y=-14$
8. $-2x-y=9$
9. $1+2y=4x$
10. $3x=4-y$
11. $x=2y-8$
12. $4x=5y$
13. $\frac{1}{2}x+\frac{1}{3}y=4$
14. $\frac{2}{3}x+\frac{1}{4}y=\frac{1}{2}$
15. $\frac{2}{7}x-\frac{1}{14}y=\frac{3}{4}$
16. $\frac{1}{6}x-\frac{2}{9}y=1$

Find the slope of the line through the following points.

17. (1, 1), (2, 3)
18. (2, 1), (-4, 2)
19. (3, 2), (-2, -3)
20. (-1, 2), (5, -4)
21. (-3, -2), (7, -4)
22. (5, 3), (-2, 3)
23. (-1, 4), (-1, -3)
24. (-9, 12), (-8, 3)
25. (-1, $\frac{1}{2}$), (-2, $\frac{1}{4}$)
26. ($\frac{1}{2}$, $-\frac{2}{3}$), ($-\frac{3}{4}$, $\frac{1}{6}$)
27. ($-\frac{5}{7}$, $\frac{1}{4}$), ($\frac{3}{14}$, $\frac{5}{6}$)

Identify the slope and use it to graph the equation.

28. $y=2x-3$
29. $y=-3x+5$
30. $y=x+1$
31. $y=\frac{1}{2}x-2$
32. $y=-\frac{3}{4}x+2$
33. $y=-\frac{2}{3}x-3$
34. $y=\frac{3}{5}x+7$
35. $y=1\frac{2}{5}x+4$
36. $2x+y=4$
37. $x+3y=6$
38. $x-y=0$
39. $2x+3y=6$
40. $4x-y=2$
41. $5x-4y=-20$
42. $x+7y=-14$
43. $-2x-y=9$

Find the equation of the line in slope-intercept form.

44. (2, 4), $m=3$
45. (3, -1), $m=-2$
46. (-2, 6), $m=\frac{1}{2}$
47. (-1, -3), $m=\frac{2}{3}$

48. (-1, -4), $m = -\frac{1}{3}$ 49. (-2, -2), $m = -\frac{3}{5}$ 50. (-2, 1), (-1, 3) 51. (3, -2), (4, 5)

52. (-2, 5), (-5, -3) 53. (4, -7), (-5, 12) 54. (-6, -1), (-1, -8) 55. $(\frac{1}{2}, -\frac{2}{3}), (-\frac{3}{4}, \frac{1}{6})$

Find the equation of the line in standard form.

56. (-2, 3), $m = -3$ 57. (4, -2), $m = 5$ 58. (3, -6), $m = \frac{1}{2}$ 59. (-2, -9), $m = -\frac{1}{3}$

60. (-2, -1), $m = -\frac{3}{5}$ 61. (-6, -5), $m = -\frac{1}{3}$ 62. (-3, -2), (7, -4) 63. (5, -3), (-2, 3)

64. (1, 4), (-1, -3) 65. (-6, 12), (-8, 3) 66. (2, $\frac{1}{2}$), (3, $\frac{1}{4}$) 67. $(\frac{1}{2}, -\frac{2}{3}), (-\frac{3}{4}, \frac{1}{6})$ 68. $(-\frac{5}{7}, \frac{1}{4}), (\frac{3}{14}, \frac{5}{6})$

Determine the equation of the line based upon the graph given. Write your answer in both slope intercept form and standard form.

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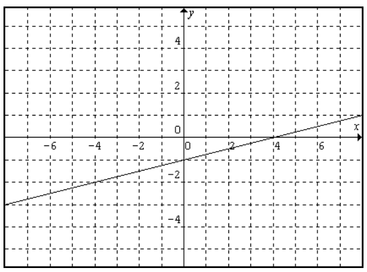

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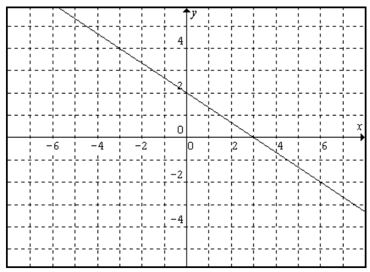

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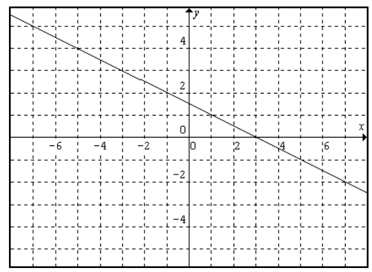

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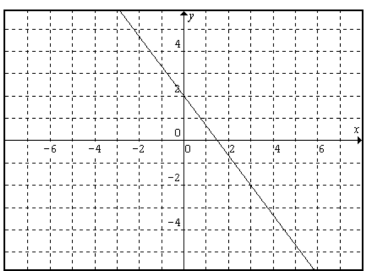

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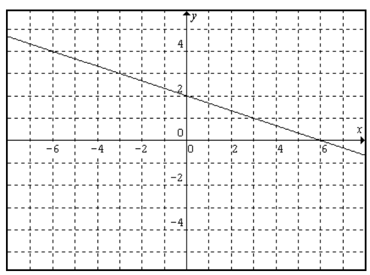

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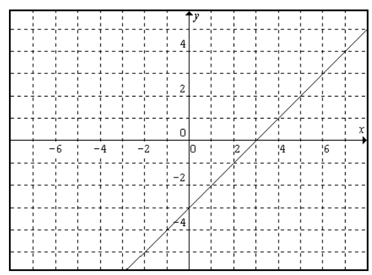

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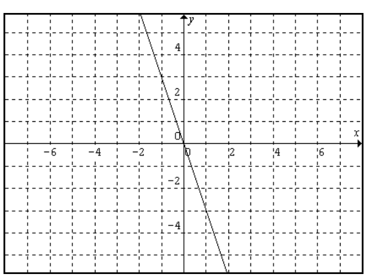

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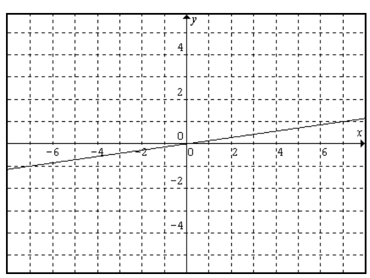

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