8.4 Multiplication of Radicals

In this section we want to learn how to multiply expressions containing radicals.

First we will need to recall the following property from section 8.2

Product Property of Radicals
If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers then, $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

For multiplying radicals we really want to look at this property as $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$. This means to multiply radicals, we simply need to multiply the coefficients together and multiply the radicands together. Then simplify as usual.

<u>Example 1</u>:

Multiply.

a. $\sqrt[3]{6} \cdot \sqrt[3]{36}$ b. $\sqrt{5x^3y} \cdot \sqrt{10x^3y^4}$ c. $\sqrt[4]{36a^2b^4} \cdot \sqrt[4]{12a^5b^3}$

Solution:

a. Using the property above, we simply multiply the radicands together and then simplify.

$$\begin{aligned}\sqrt[3]{6} \cdot \sqrt[3]{36} &= \sqrt[3]{216} \\ &= 6\end{aligned}$$

b. Just as above we multiply the radicands and simplify.

$$\begin{aligned}\sqrt{5x^3y} \cdot \sqrt{10x^3y^4} &= \sqrt{50x^6y^5} \\ &= \sqrt{25x^6y^4(2y)} \\ &= 5x^3y^2\sqrt{2y}\end{aligned}$$

c. Again, we proceed as above.

$$\begin{aligned}\sqrt[4]{36a^2b^4} \cdot \sqrt[4]{12a^5b^3} &= \sqrt[4]{432a^7b^7} \\ &= \sqrt[4]{2^4a^4b^4(3^3a^3b^3)} \\ &= 2ab\sqrt[4]{27a^3b^3}\end{aligned}$$

Now, in order to multiply expressions containing more that one term, we will simply multiply as we did with polynomials in the past. That is, we multiply each term in the first expression by each term in the second expression, and simplify.

<u>Example 2</u>:

Multiply.

a. $\sqrt{y}\left(\sqrt{y} - \sqrt{5}\right)$ b. $\left(\sqrt{2x} + 4\right)^2$

c. $\left(\sqrt{14} - 3\right)\left(\sqrt{2} + 4\right)$ d. $\left(\sqrt{2x} - 3\sqrt{y}\right)\left(\sqrt{2x} + 3\sqrt{y}\right)$

Solution:

a. Since we are to multiply as we did with polynomials, we need to use the distrubutive property here. We must always keep in mind, though, that to multiply radicals we multiply the radicands. So we get

$$\begin{aligned}\sqrt{y}\left(\sqrt{y}-\sqrt{5}\right) &= \sqrt{y}\sqrt{y}-\sqrt{y}\sqrt{5}\\ &= \sqrt{y^2}-\sqrt{5y}\\ &= y-\sqrt{5y}\end{aligned}$$

b. In this example we have to remember that we cannot pull an exponent though a set of parenthesis if the operation inside is addition or subtraction. Instead we need to write out binomial twice and then multiply out as we did before. That is with either the FOIL method or multiplying each term in the first expression by each term in the second. We get

$$\begin{aligned}\left(\sqrt{2x}+4\right)^2 &= \left(\sqrt{2x}+4\right)\left(\sqrt{2x}+4\right)\\ &= \sqrt{2x}\sqrt{2x}+4\sqrt{2x}+4\sqrt{2x}+4\cdot 4\\ &= \sqrt{4x^2}+8\sqrt{2x}+16\\ &= 2x+8\sqrt{2x}+16\end{aligned}$$

c. This time we simply proceed like we did in part b. Multiply each term in the first by each term in the second. This gives

$$\begin{aligned}\left(\sqrt{14}-3\right)\left(\sqrt{2}+4\right) &= \sqrt{14}\sqrt{2}+4\sqrt{14}-3\sqrt{2}-3\cdot 4\\ &= \sqrt{28}+4\sqrt{14}-3\sqrt{2}-12\\ &= 2\sqrt{7}+4\sqrt{14}-3\sqrt{2}-12\end{aligned}$$

d. Again, proceed like we did above.

$$\begin{aligned}\left(\sqrt{2x}-3\sqrt{y}\right)\left(\sqrt{2x}+3\sqrt{y}\right) &= \sqrt{2x}\sqrt{2x}+\sqrt{2x}\cdot 3\sqrt{y}-3\sqrt{y}\sqrt{2x}-3\sqrt{y}\cdot 3\sqrt{y}\\ &= \sqrt{4x^2}+3\sqrt{2xy}-3\sqrt{2xy}-9\sqrt{y^2}\\ &= 2x-9y\end{aligned}$$

Notice in the very last example that the answer ended up with no radicals. We call it a "radical free" expression. We also notice that the two expressions $\sqrt{2x}-3\sqrt{y}$ and $\sqrt{2x}+3\sqrt{y}$ only differ by the sign between the terms. When this is the case we call the expressions conjugates.

Definition: Conjugates
The expressions $a\sqrt{b}+c\sqrt{d}$ and $a\sqrt{b}-c\sqrt{d}$ are called <u>conjugates</u>. Conjugates always have a product that is "radical free".

We will need conjugates in an important way in the next section. For now we simply need to remember that they always multiply to give an expression containing no radicals.

8.4 Exercises

Multiply.

1. $\sqrt{2}\sqrt{8}$
2. $\sqrt{3}\sqrt{27}$
3. $\sqrt{x^2y^5}\sqrt{xy}$
4. $\sqrt{a^2b}\sqrt{ab^4}$
5. $\sqrt[3]{4x}\sqrt[3]{8x}$
6. $\sqrt{14y}\sqrt{35y}$
7. $\sqrt[3]{9t^2}\sqrt[3]{9t^2}$
8. $\sqrt[3]{18x}\sqrt[3]{4x^4}$
9. $\sqrt[3]{16x^4}\sqrt[3]{4x}$
10. $\sqrt[3]{10a^5}\sqrt[3]{50a^3}$
11. $\sqrt{5x^3y}\sqrt{10x^3y^4}$
12. $\sqrt{2x^2y}\sqrt{32xy}$
13. $\sqrt[3]{9x^4y^2}\sqrt[3]{18xy}$
14. $\sqrt[3]{16xy^2}\sqrt[3]{12x^2y}$
15. $\sqrt[5]{27n^4m}\sqrt[5]{9n^2m^4}$
16. $\sqrt[3]{4ab^2}\sqrt[3]{18a^8b}$
17. $\sqrt[4]{18a^3b^5}\sqrt[4]{6a^5b^3}$
18. $\sqrt[5]{8x^3y^4}\sqrt[5]{8x^4y^6}$
19. $\sqrt[4]{27a^2b^3}\sqrt[4]{9ab}$
20. $\sqrt[4]{8x^6y}\sqrt[4]{6x^2y^3}$
21. $\sqrt{y}\left(\sqrt{y}+4\right)$
22. $\sqrt{2x}\left(\sqrt{x}-3\right)$
23. $\sqrt{3}\left(\sqrt{3}-\sqrt{4}\right)$
24. $\sqrt{12}\left(\sqrt{3}-\sqrt{12}\right)$
25. $2\sqrt{y}\left(\sqrt{2x}-\sqrt{y}\right)$
26. $\sqrt{ab}\left(a\sqrt{b}+3\sqrt{b}\right)$
27. $\left(\sqrt{5x}+6\right)^2$
28. $\left(\sqrt{x}+\sqrt{y}\right)^2$
29. $\left(4-\sqrt{20}\right)^2$
30. $\left(\sqrt{a}+2\sqrt{b}\right)^2$
31. $\left(x\sqrt{y}+\sqrt{z}\right)^2$
32. $\left(\sqrt{2a}+\sqrt{2b}\right)^2$
33. $\left(\sqrt{a}-\sqrt{4b}\right)^2$
34. $\left(\sqrt{9x}+1\right)^2$
35. $\left(\sqrt{x}+2\right)\left(2\sqrt{x}-3\right)$
36. $\left(2-\sqrt{a}\right)\left(7+4\sqrt{a}\right)$
37. $\left(3+\sqrt{2}\right)\left(2-\sqrt{8}\right)$
38. $\left(1-\sqrt{27}\right)\left(1+\sqrt{3}\right)$
39. $\left(\sqrt{\alpha}+\beta\sqrt{\chi}\right)\left(\sqrt{\alpha}-2\beta\sqrt{\chi}\right)$
40. $\left(3\sqrt{n}+4\sqrt{m}\right)\left(\sqrt{n}-\sqrt{m}\right)$
41. $\left(2\sqrt{p}-3\sqrt{q}\right)\left(\sqrt{p}+2\sqrt{q}\right)$
42. $\left(7\sqrt{x}-3\sqrt{y}\right)\left(2\sqrt{x}+\sqrt{y}\right)$
43. $\left(\sqrt{a}+2\sqrt{b}\right)\left(\sqrt{a}-3\sqrt{b}\right)$
44. $\left(2\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}-2\sqrt{y}\right)$
45. $\left(\sqrt{x}+\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)$
46. $\left(\sqrt{x}+5\right)\left(\sqrt{x}-5\right)$
47. $\left(2\sqrt{a}-\sqrt{b}\right)\left(2\sqrt{a}+\sqrt{b}\right)$
48. $\left(5\sqrt{b}-3\sqrt{a}\right)\left(5\sqrt{b}+3\sqrt{a}\right)$
49. $\left(\sqrt{x}+1\right)\left(x+\sqrt{x}-1\right)$
50. $\left(\sqrt{a}-1\right)\left(a-\sqrt{a}+1\right)$
51. $\left(a+\sqrt{b}+\sqrt{a}\right)\left(\sqrt{a}+\sqrt{b}\right)$
52. $\left(2a+b-\sqrt{a}\right)\left(\sqrt{a}-1\right)$
53. $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}+\sqrt{xy}\right)$
54. $\left(\sqrt{a}+\sqrt{2}\right)\left(\sqrt{a}+\sqrt{2}-1\right)$
55. $\left(\sqrt[3]{y}+2\right)\left(\sqrt[3]{y^2}-5\right)$
56. $\left(\sqrt[3]{a^2}+1\right)\left(\sqrt[3]{a}-3\right)$
57. $\left(\sqrt[3]{x}-2\right)\left(1+5\sqrt[3]{x^2}\right)$
58. $\left(\sqrt[3]{y^2}+3\right)\left(1-\sqrt[3]{y}\right)$
59. $\left(\sqrt[4]{2x}-1\right)\left(\sqrt[4]{8x^3}-2\right)$
60. $\left(\sqrt[4]{4a^2}+1\right)\left(\sqrt[4]{4a^2}-1\right)$
61. $\left(\sqrt{x+1}+2\right)^2$
62. $\left(\sqrt{2x-1}-3\right)^2$
63. $\left(\sqrt{x}-\sqrt{x-2}\right)^2$
64. $\left(\sqrt{x-3}+\sqrt{2x}\right)^2$
65. $\left(\sqrt{3x-1}-\sqrt{2x+1}\right)^2$
66. $\left(\sqrt{x+4}+\sqrt{5x-1}\right)^2$
67. $\left(\sqrt{2x}+\sqrt{2x-1}\right)^2$
68. $\left(\sqrt{4x+2}-\sqrt{2x+3}\right)^2$