8.3 Addition and Subtraction of Radicals

In this section we want to learn how to add and subtract radicals.

Adding and subtracting radicals is done in the same manner as polynomials are. That is we simply need to "combine like radicals". Of course we need to define what it means for two radicals to be like radicals.

Definition: Like Radicals
Two radicals are <u>like radicals</u> if they have the same index and same radicand. That is, if everything after the coefficient is exactly the same, then the radicals are like.

Of course, in order to tell if two radicals are like, they first need to be in simplest form.

So to combine like radicals we just add or subtract coefficients, just as we did for polynomials.

<u>Example 1</u>:

Perform the indicated operations.

a. $3\sqrt{y} + 12\sqrt{y}$

b. $\sqrt{18b} - \sqrt{75b}$

c. $2\sqrt{32x^2 y^3} - xy\sqrt{98y}$

d. $2\sqrt[3]{24x^3 y^4} + 4x\sqrt[3]{81y^4} - 3y\sqrt[3]{24x^3 y}$

Solution:

a. Notice first that these are like radicals. So we simply need to add the coefficients to get
$$3\sqrt{y} + 12\sqrt{y} = 15\sqrt{y}$$

b. First we need to simplify each of the radicals as we learned in the last section.
$$\sqrt{18b} - \sqrt{75b} = \sqrt{9 \cdot 2b} - \sqrt{25 \cdot 3b}$$
$$= 3\sqrt{2b} - 5\sqrt{3b}$$
Since the radicand is different, the radicals are not like. Therefore we cannot combine them and we are finished with the problem.

c. Again, we will simplify the radicals and combine them if they are like.
$$2\sqrt{32x^2 y^3} - xy\sqrt{98y} = 2\sqrt{16x^2 y^2 (2y)} - xy\sqrt{49(2y)}$$
$$= 2 \cdot 4xy\sqrt{2y} - 7xy\sqrt{2y}$$
$$= 8xy\sqrt{2y} - 7xy\sqrt{2y}$$
$$= xy\sqrt{2y}$$

d. Lastly, we proceed as we did above, simplifying and combining.
$$2\sqrt[3]{24x^3 y^4} + 4x\sqrt[3]{81y^4} - 3y\sqrt[3]{24x^3 y} = 2\sqrt[3]{8x^3 y^3 (3y)} + 4x\sqrt[3]{27y^3 (3y)} - 3y\sqrt[3]{8x^3 (3y)}$$
$$= 4xy\sqrt[3]{3y} + 12xy\sqrt[3]{3y} - 6xy\sqrt[3]{3y}$$
$$= 10xy\sqrt[3]{3y}$$

8.3 Exercises

Perform the indicated operations.

1. $3\sqrt{2} + 5\sqrt{2}$

2. $5\sqrt{3} + 8\sqrt{3}$

3. $9\sqrt{15} - \sqrt{15}$

4. $6\sqrt[3]{5} - 7\sqrt[3]{5}$

5. $2x\sqrt[3]{y^2} + 5x\sqrt[3]{y^2}$

6. $t\sqrt{m} - 2t\sqrt{m}$

7. $5\sqrt[4]{12x^2} - 7\sqrt{x} + 8\sqrt[4]{12x^2}$

8. $6y\sqrt[3]{x} - y\sqrt[3]{x} + 2\sqrt[3]{x}$

9. $9\sqrt[3]{17} + 7\sqrt[3]{2} - 4\sqrt[3]{17} - \sqrt[3]{2}$

10. $5\sqrt{30} - 2\sqrt{2} + 2\sqrt{30} - 6\sqrt{2}$

11. $\sqrt{8} + \sqrt{18}$

12. $\sqrt{27} + 2\sqrt{12}$

13. $\sqrt[3]{54x} - \sqrt[3]{16x}$

14. $2\sqrt[3]{6x^2 y} - \sqrt[3]{48x^2 y}$

15. $x\sqrt{9xy^2} + 4\sqrt{x^3 y^2}$

16. $5b\sqrt{18a^2 b} + a\sqrt{8b^3}$

17. $\sqrt[4]{48} - \sqrt[4]{243}$

18. $\sqrt[4]{32t^5} - t\sqrt[4]{192t}$

19. $\sqrt[3]{16t^4} - t\sqrt[3]{54t}$

20. $\sqrt[3]{6x^5} - \sqrt[3]{48x^5}$

21. $a\sqrt{5a} + 3\sqrt{45a^3}$

22. $5\sqrt{3x^3} - 3\sqrt{12x^3}$

23. $5\sqrt{8x^2 y^3} - 6x\sqrt{32y^3}$

24. $\sqrt{32a^2 b^3} - ab\sqrt{98b}$

25. $4a\sqrt{27ab^5} + 5b\sqrt{3a^3 b}$

26. $3\sqrt[3]{3a^4} - 2a\sqrt[3]{81a}$

27. $3b\sqrt[3]{16b^5} + \sqrt[3]{128b^8}$

28. $2\sqrt[3]{x^5 y^9} - 6xy\sqrt[3]{x^2 y^4}$

29. $\sqrt[4]{32a^5} - a\sqrt[4]{162a}$

30. $7x\sqrt[4]{16xy^5} + 5y\sqrt[4]{256x^5 y}$

31. $3\sqrt{50} - 4\sqrt{125} - 2\sqrt{98}$

32. $17\sqrt{108} - 5\sqrt{18} - 6\sqrt{48}$

33. $3\sqrt{50a^2} - 3\sqrt{72a^2} - 8a\sqrt{18}$

34. $3\sqrt{27c^2} - 2\sqrt{108c^2} - \sqrt{48c^2}$

35. $\sqrt{4r^7 s^5} + 3r^2\sqrt{r^3 s^5} - 2rs\sqrt{r^5 s^3}$

36. $x\sqrt[3]{27x^5 y^2} - x^2\sqrt[3]{x^2 y^2} + 2\sqrt{x^8 y^2}$

37. $\sqrt[3]{4a^7 b^5} - 8a\sqrt[3]{a^4 b^5} - 4ab\sqrt[3]{a^4 b^2}$

38. $\sqrt{49y^3} - \sqrt{9y^3} - \sqrt{25y^3}$

39. $7x\sqrt{8xy^2} - 4y\sqrt{32x^3} + 9\sqrt{4x^2 y^3}$

40. $4x\sqrt{3x^3 y} + 5x^2\sqrt{27xy} - 10\sqrt{75x^5 y}$

41. $2\sqrt[3]{54xy^3} - 3\sqrt[3]{2xy^3} + y\sqrt[3]{128x}$

42. $2\sqrt[3]{24x^3 y^4} + 6x\sqrt[3]{81y^4} - 6y\sqrt[3]{24x^2 y}$

43. $a\sqrt[3]{32b^5} - 3b\sqrt[3]{162a^4 b} + \sqrt[3]{2a^4 b^5}$

44. $12\sqrt[4]{48x^5} - 10x\sqrt[4]{243xy^4} - 4\sqrt[4]{3x^5 y^4}$

45. $3y\sqrt[4]{48x^5} - x\sqrt[4]{3x^5 y^4} - xy\sqrt[3]{3x}$

46. $3 \cdot \sqrt[3]{24x^4 y} + x \cdot \sqrt[3]{192xy} - \sqrt{375x^4 y}$

47. $4x \cdot \sqrt[3]{24x} + 3 \cdot \sqrt[3]{81x^4} - \sqrt{9x^4}$

48. $\sqrt[3]{128x^9 y^{10}} - 2x^2 y\sqrt[3]{16x^3 y^7} + \sqrt{2x^6 y^5}$

49. $5\sqrt[3]{320x^5 y^8} - 2x\sqrt[3]{135x^2 y^8} - xy^2\sqrt{5x^2 y^2}$

50. $5u \cdot \sqrt[3]{24u^2} + 2 \cdot \sqrt[3]{81u^5} - 3u \cdot \sqrt{3u^2}$