# 7.5 Systems of Linear Inequalities

In chapter 6 we discussed linear equations and then saw linear inequalities, so since here, in chapter 7, we are talking about systems of linear equations, it makes sense to talk about a system of inequalities.

**Definition**: **System of Linear Inequalities**- Two or more linear inequalities involving the same variables.

As usual, solutions to systems are ordered pairs that satisfy every inequality in the system.

As we did in chapter 6, we will need to graph these inequalities in order to make sense of the solutions. If we recall, the solutions to a single inequality was a shaded half-plane.

So, for a system of inequalities, the solution will simply be the region on the rectangular coordinate system where the shading of the inequalities overlap. Let's see some examples to clarify this idea.

Example 1:

Graph the system of inequalities.

a. $\begin{array}{c} x-3y<-3 \\ 3x+y\le -2 \end{array}$    b. $\begin{array}{c} 3x-y\ge -10 \\ 3x+y<4 \end{array}$

Solution:

a. First, we need to graph both inequalities as on the same plane. We learned how to graph these in chapter 6. So we start by finding all of the information we did before.

$\underline{x-3y<-3:}$

$$\begin{aligned} x-3y&=-3 \\ -x \quad &\quad -x \\ -3y&=-x-3 \\ y&=\tfrac{1}{3}x+1 \end{aligned}$$

m = $\frac{1}{3}$, y-int: (0, 1)

$\underline{3x+y\le -2:}$

$$\begin{aligned} 3x+y&=-2 \\ -3x \quad &\quad -3x \\ y&=-3x-2 \end{aligned}$$

m = -3, y-int: (0, -2)

Then, our x-intercepts for check points.

$$\begin{aligned} x-3y&=-3 \\ x-3(0)&=-3 \\ x&=-3 \end{aligned}$$

$$\begin{aligned} 3x+y&=-2 \\ 3x+0&=-2 \\ x&=-\tfrac{2}{3} \end{aligned}$$

And we will need a test point to determine shading. Let's use (0, 0)

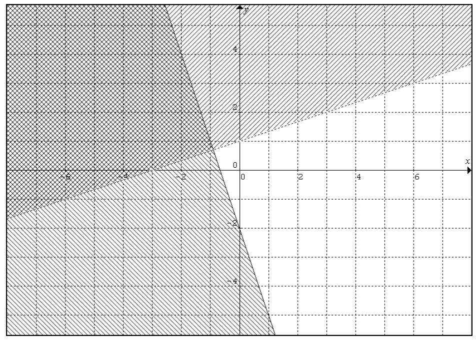
$$\begin{aligned} x-3y&<-3 \\ 0-3(0)&<-3 \\ 0&<-3 \end{aligned}$$

False

$$\begin{aligned} 3x+y&\le -2 \\ 3(0)+0&\le -2 \\ 0&\le -2 \end{aligned}$$

False

Recall that if the inequality is < or > we use a solid line, if it is ≤ or ≥ it is a dotted line. So graphing together we get



The solution to the system is only the region that is "double shaded."

b. Again, here we start with graphing the inequalities on the same axis. The work for this is below.

$\underline{3x - y \geq -10:}$
$3x - y = -10$
$-3x \quad\quad - 3x$
$-y = -3x - 10$
$y = 3x + 10$
m =3 , y-int: (0, 10)

$\underline{3x + y < 4:}$
$3x + y = 4$
$-3x \quad\quad - 3x$
$y = -3x + 4$
m = -3, y-int: (0, 4)

Then, our x-intercepts for check points.

$3x - y = -10$
$3x - 0 = -10$
$x = -\frac{10}{3}$

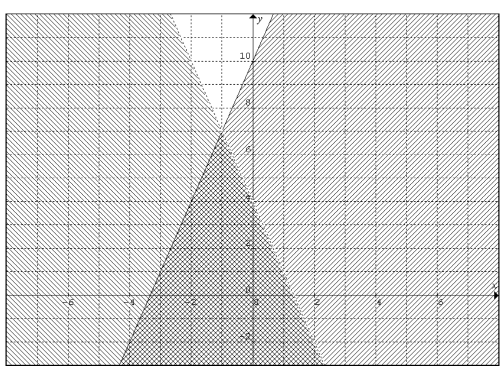
$3x + y = 4$
$3x + 0 = 4$
$x = \frac{4}{3}$

And we will need a test point to determine shading. Let's use (0, 0)

$3x - y \geq -10$
$3(0) - 0 \geq -10$
$0 \geq -10$
True

$3x + y < 4$
$3(0) + 0 < 4$
$0 < 4$
True

Putting it together we get



Again, the region shaded by both is the solution to the system.

Of course, if you recall, systems of equations can have some special cases, namely no solution and infinite solutions.

For inequalities, if the associated lines end up to being parallel, it would depend on the shading to determine if the system has no solution.

If the associated lines end up being the same line, you could end up with a number of things happening, from no solution, to just the line, to an entire half of the rectangular plane.  Its best to just see what happens as they come up.

<u>Example 2</u>:

Graph the system of inequalities.

a. $\begin{matrix} 2x + y > 3 \\ y \leq -2x - 6 \end{matrix}$   b. $\begin{matrix} x - y \leq 4 \\ 2x - 2y \geq 8 \end{matrix}$   c. $\begin{matrix} x + 2y > 2 \\ 3x + 6y \geq 6 \end{matrix}$

Solution:

a. So we start, again, by finding all of our usual information to find the graphs.

$\underline{2x + y > 3:}$
$2x + y = 3$
$-2x \qquad -2x$
$y = -2x + 3$
m =-2, y-int: (0, 3)

$\underline{y \leq -2x - 6:}$
$y = -2x - 6$
m = -2, y-int: (0, -6)

Since we have the same slope but different y-intercepts, the lines must be parallel.  Lets check with our x-intercepts.

$2x + y = 3$
$2x + 0 = 3$
$x = \frac{3}{2}$

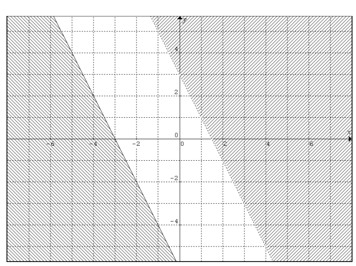
$y = -2x - 6$
$0 = -2x - 6$
$+6 \qquad +6$
$6 = -2x$
$x = -3$

This again verifies that we must have parallel lines and not the same line.
In any case, we will need a test point to determine shading.  Let's use (0, 0)

$2x + y > 3$
$2(0) + 0 > 3$
$0 > 3$
False

$y \leq -2x - 6$
$0 \leq -2(0) - 6$
$0 \leq -6$
False

Now let's graph them.



Since clearly the shaded regions do not overlap, this system has no solution.

b. First, we find the information we need for the graphs.

| | |
|---|---|
| $\underline{x - y \leq 4:}$ | $\underline{2x - 2y \geq 8:}$ |
| $x - y = 4$ | $2x - 2y = 8$ |
| $-x \qquad -x$ | $-2x \qquad -2x$ |
| $-y = -x + 4$ | $-2y = -2x + 8$ |
| $y = x - 4$ | $y = x - 4$ |
| m =1, y-int: (0, -4) | m =1, y-int: (0, -4) |

Since we have the same slope and same y-intercepts, the lines must be the same line. Let's check with our x-intercepts.

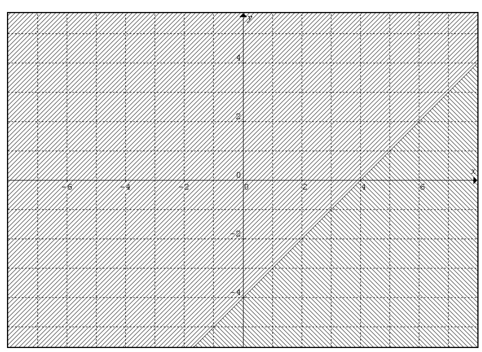
| | |
|---|---|
| $x - y = 4$ | $2x - 2y = 8$ |
| $x - 0 = 4$ | $2x - 2(0) = 8$ |
| $x = 4$ | $2x = 8$ |
| | $x = 4$ |

Obviously they are the same line. However, we still need to be concerned about how the shading of the lines interacts with each other. So we still test each inequality and graph to see what we have.

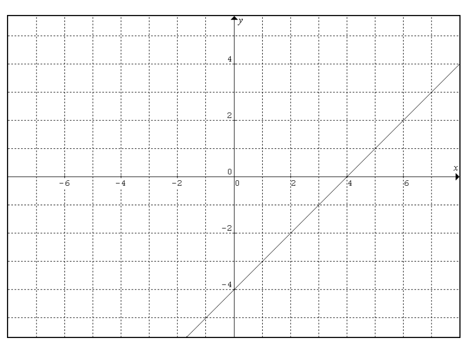
Testing (0, 0)

| | |
|---|---|
| $x - y \leq 4$ | $2x - 2y \geq 8$ |
| $0 - 0 \leq 4$ | $2(0)-2(0)\geq 8$ |
| $0 \leq 4$ | $0 \geq 8$ |
| True | False |

Now let's graph and see what we can deduce.



Since the "shading" part does not overlap, but the lines are both solid, the solution to the system is only the line. So the solution is just the line



c. Finally, we get all of our standard information so that we can graph.

$\underline{x + 2y > 2:}$
$x + 2y = 2$
$-x \quad\quad -x$
$2y = -x + 2$
$y = -\frac{1}{2}x + 1$
m =- ½ , y-int: (0, 1)

$\underline{3x + 6y \geq 6:}$
$3x + 6y = 6$
$-3x \quad\quad -3x$
$6y = -3x + 6$
$y = -\frac{1}{2}x + 1$
m =- ½ , y-int: (0, 1)

Again we have the same slope and same y-intercepts, the lines must be the same line. Let's check with our x-intercepts.

$x + 2y = 2$
$x + 2(0) = 2$
$x = 2$

$3x + 6y = 6$
$3x + 6(0) = 6$
$3x = 6$
$x = 2$

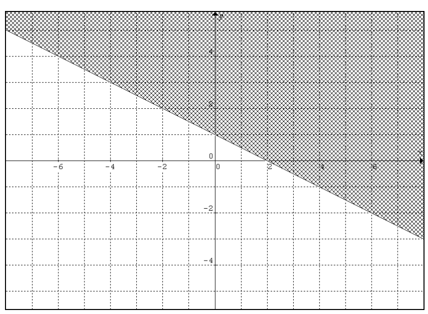
Obviously they are the same line. However, we still need to be concerned about how the shading of the lines interacts with each other. So we still test each inequality and graph to see what we have.

Testing (0, 0)

$x + 2y > 2$
$0 + 2(0) > 2$
$0 > 2$
False

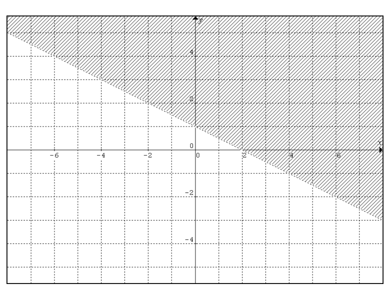
$3x + 6y \geq 6$
$3(0) + 6(0) \geq 6$
$0 \geq 6$
False

Now let's graph and see what we can deduce.



In this case, the shading overlaps everywhere, however, one line is dotted and one is solid. Since, to be a solution to a system, an ordered pair must be a solution to all inequalities of the system, the dotted line must be the one that we use (the points on the line are not solutions to both inequalities).

So our solution is only the graph of x + 2y > 2



## 7.5 Exercises

Graph the system of inequalities.

1. $y > 6x - 11$; $2x + 3y < 7$

2. $7x + 2y > -13$; $x - 2y \geq 11$

3. $2x - 3y < -1$; $y \leq x - 1$

4. $x + 2y \leq -4$; $4y \geq 3x + 12$

5. $y \geq -3x + 5$; $5x - 4y \leq -3$

6. $x + y < 0$; $3x + y > -4$

7. $3x + 3y < -3$; $y > -5x - 17$

8. $x - 3y \leq 6$; $2x - 6y \geq 6$

9. $6x - 4y > 2$; $3x - 2y < 4$

10. $x + 3y \leq 3$; $2x - y \leq 6$

11. $y > 2x - 1$; $2x - 3y \leq 2$

12. $6y > 2x - 12$; $x - 3y \leq 6$

13. $y < -2$; $4x - 3y > 18$

14. $y < 1$; $2x + 3y < 3$

15. $y \leq 5x - 7$; $3x + 2y \geq 12$

16. $3x + 8y \geq 20$; $-5x + y < 19$

17. $-5x + y < -2$; $3x - 6y \geq 12$

18. $2x + y \geq 1$; $x - y < 5$

19. $-4x + y < 6$; $5x + y < -21$

20. $2x - y > 3$; $-4x + 2y \geq -6$

21. $5x - y > 3$; $3x - 8y \geq 24$

22. $y < 2x$; $x + y \geq 3$

23. $x + 3y \geq 1$; $2x + 6y \leq 2$

24. $3x + 4y > -2$; $3x + 3y > -3$

25. $3x - 3y < 4$; $x - y < -3$

26. $y > x + 2$; $4x + 3y < -15$

27. $6x + 6y < -6$; $5x + y > -13$

28. $2x + y \leq 9$; $5x - 2y \leq 18$

29. $y \geq x + 1$; $6x + 3y \geq 3$

30. $3y \leq 2x - 3$; $y \geq -x + 4$

31. $x < 2$; $2x + 3y \geq 4$

32. $2y \geq 6x - 4$; $3x - y < 7$

33. $y \leq x - 3$; $2x - 2y < 7$