7.4 Applications of Linear Systems

Now that we can solve linear systems using a variety of techniques, there are several different kinds of applications that we want to take a look at.

There are two main types of applications that we want to investigate: Moving Objects (which we have seen before) and Mixture (which has two different types).

To do these, we will need some formulas.

Moving Objects: $r \cdot t = d$

Mixture:

Involving money: $Amount \cdot Cost = Total\ Value$ or $A \cdot C = TV$

Involving Percent: $Amount \cdot Rate = Quantity$ or $A \cdot r = Q$

As we did in chapter 5, we use a chart to produce our equations.

However, on the mixture problems, whenever the ending mixture is given in terms of a percent, we need to add a row in the chart for the final mixture.

Example 1:

At a barbecue, there were 250 dinners served. Children's plates were $1.50 each and adult's plates were $2.00 each. If the total amount of money collected for dinners at the barbecue was $441, how many of each type of plate was served?

Solution:

The first thing we need to do is identify the type of problem. Clearly, this is a mixture problem which involves money. So we will have to use the formula $A \cdot C = TV$. Next we see that we are "mixing" together adult plates and children's plates. So we start to build our chart and fill in the information in the problem.

We can see that the cost of a child plate is $1.50 and an adult plate is $2. So we will into the chart what information we know so far. It gives us

	Amount	Cost	Total Value
Children		1.50	
Adults		2	

Since we are looking to find out how many children's plates and adult plates were sold, we can label those with our variables, x and y, and then use the formula fill in the chart by multiplying straight across. We get

	Amount	Cost	Total Value
Children	x	1.5	1.5x
Adults	y	2	2y

Now with our chart filled in, we simply need to generate the equations. To do so, we look back at the problem. Since we know that there was 250 total dinners served, this means we must have the equation $x + y = 250$. Also, according to the chart, the total value of all the children's plates is 1.5x and the total value of all the adult plates is 2y, and the total money collected was $441, we must have $1.5x + 2y = 441$.

Therefore our system is

$$x + y = 250$$
$$1.5x + 2y = 441$$

Now we can solve using any technique that we wish. Usually, the elimination method works best, however, either elimination or substitution will result in the same answer. Let's use elimination.

We start by multiplying the top equation by -2 to eliminate the y's.

$$\begin{array}{c} -2(x + y = 250) \\ 1.5x + 2y = 441 \end{array} \longrightarrow \begin{array}{rcl} -2x - 2y &=& -500 \\ 1.5x + 2y &=& 441 \\ \hline -0.5x \quad &=& -59 \end{array}$$

Dividing by -0.5 on both sides gives

$$-0.5x = -59$$
$$\frac{-0.5x}{-0.5} = \frac{-59}{-0.5}$$
$$x = 118$$

Now we find the y by plugging the 118 into the top original equation and solve.

$$x + y = 250$$
$$118 + y = 250$$
$$-118 \qquad -118$$
$$y = 132$$

Since x was the number of children's plates and y was the number of adult plates, we have 118 children's plates and 132 adult plates.

Example 2:

In his prime Shaquille O'Neal was one of the most dominate basketball players in history. He was also one of the worst free throw shooters. If Shaq's highest single game total is 61 points which he earned with 37 combined 2 point field goals and 1 point free throws, how many of each type of basket did he score?

Solution:

This time we notice that the example doesn't seem to fit into any one of the formulas. It turns out we can use the "points" in the problem as we would the "cost" in a money problem. So, even though it's not a mixture problem involving money, we will still use the $A \cdot C = TV$.

So, here, we are mixing together different types of basketball shots, the 2 point field goal and the 1 point free throw. So we get our chart as below.

	Amount	Cost	Total Value
2 pt field goal		2	
1 pt free throw		1	

Now putting x for the amount of 2 point field goals and y for the amount of 1 point free throws, then multiplying straight across gives

	Amount	Cost	Total Value
2 pt field goal	x	2	2x
1 pt free throw	y	1	y

So, to get the equations we need to see that the total combined shots that Shaquille made was 37, which gives us x + y = 37 and the total point value he earned was 61 points, which gives us 2x + y = 61. So our system is

$$x + y = 37$$
$$2x + y = 61$$

Since we solved the last example by elimination, lets solve this one by substitution, just so that we can see that the method we use true doesn't matter.

So we start by solving the top equation for y and substituting it into the other equation.

$$\begin{aligned} x + y &= 37 \\ -x \quad &\quad -x \\ y &= -x + 37 \end{aligned}$$

Substituting gives

$$\begin{aligned} 2x + y &= 61 \\ 2x + (-x + 37) &= 61 \\ x + 37 &= 61 \\ -37 \quad &\quad -37 \\ x &= 24 \end{aligned}$$

Plugging x = 24 back in we get

$$\begin{aligned} y &= -x + 37 \\ y &= -(24) + 37 \\ y &= 13 \end{aligned}$$

So, since x is the number of 2 point shots and y is the number of 1 point shots, the answer is Shaquille made 24, 2 point field goals and 13, 1 point free throws.

Example 3:

How many grams of silver that is 60% pure must be mixed together with silver that is 35% pure to obtain a mixture of 90 grams of silver that is 45% pure?

Solution:

Here, clearly, we are dealing with a mixture problem involving percent's. So we will use the formula $A \cdot r = Q$. Also, notice the final mixture is given in terms of a percentage as well. So we will have to add a row at the bottom of the chart for this final mixture. So initially the chart looks like

	Amount	Rate	Quantity
60% pure			
35% pure			
Final: 45% pure			

As it turns out, the "rate" part of these problems is simply the decimal version of each percent in the problem. So we can put these values in the chart.

Also, we have been given the amount of 45% pure that we want to have in the end, 90 grams, so we can put that into the chart as well as variables (x and y) for our missing amounts.

	Amount	Rate	Quantity
60% pure	x	0.6	
35% pure	y	0.35	
Final: 45% pure	90	0.45	

Now we multiply across to complete the chart.

	Amount	Rate	Quantity
60% pure	x	0.6	0.6x
35% pure	y	0.35	0.35y
Final: 45% pure	90	0.45	40.5

The way we get from the chart to the equations is simple. First, note that the amounts have to all add up. That is, when you mix x grams of one silver, with y grams of another (that is x + y) you don't lose or gain any silver in the process. So from the first column we have x + y = 90.

Next, the quantity column represents the amount of actual silver in each sample. So, since we, again, wouldn't gain or lose any silver in the mixing process, from the last column we must have 0.6x + 0.35y = 40.5.

As it turns out, this is always the case with the charts with a "final" row. The top values in the first column must always add to the bottom value, and the same is true for the last column (the top two add to equal the bottom value).

So our system is

$$\begin{gathered} x + y = 90 \\ 0.6x + 0.35y = 40.5 \end{gathered}$$

In this case, since the question only wants to know about the amount of 60% pure silver, we should solve by elimination by getting rid of the y values, only leaving us with x, which is the variable associated with the 60% pure silver.

We multiply the top equation by -0.35 and solve

$$\begin{gathered} -0.35(x + y = 90) \\ 0.6x + 0.35y = 40.5 \end{gathered} \longrightarrow \begin{array}{r} -0.35x - .035y = -31.5 \\ \underline{\quad 0.6x + 0.35y = 40.5} \\ 0.25x \qquad\qquad = 9 \end{array}$$

Divide by 0.25

$$\begin{gathered} 0.25x = 9 \\ \frac{0.25x}{0.25} = \frac{9}{0.25} \\ x = 36 \end{gathered}$$

So we need 36 grams of 60% pure silver.

Example 4:

A farmer needs to make 320 pounds of feed that is 53% corn. However, he only has feed that is 80% corn and 44% corn available. How much of each should he mix to get the feed that he wants?

Solution:

Just like in Example 3, we need to use the formula $A \cdot r = Q$ and the "final" row to solve this problem. So, again putting our initial values into the chart and filling in variables and multiplying the rows gives us

	Amount	Rate	Quantity
80% corn	x	0.8	0.8x
44% corn	y	0.44	0.44y
Final: 53% corn	320	0.53	169.6

So, as we stated above, the first equation comes from adding the top two values in the first column and setting them equal to the bottom values in the first column, and the second equation comes from doing the same thing in the last column. So our equations are

$$x + y = 320$$
$$0.8x + 0.44y = 169.6$$

Once again we can solve the system using whatever method we choose. Elimination seems like the best choice again. Let's eliminate the x's.

$$\begin{array}{c} -0.8(x + y = 320) \\ 0.8x + 0.44y = 169.6 \end{array} \longrightarrow \begin{array}{r} -0.8x - 0.8y = -256 \\ \underline{0.8x + 0.44y = 169.6} \\ -0.36y = -86.4 \end{array}$$

Dividing by -0.36 gives

$$\frac{-0.36y}{-0.36} = \frac{-86.4}{-0.36}$$

$$y = 240$$

Subbing in to find x gives

$$\begin{array}{r} x + y = 320 \\ x + 240 = 320 \\ -240 \quad -240 \\ x = 80 \end{array}$$

So the farmer needs 80 pounds of 80% corn feed and 240 pounds of 44% corn feed.

Example 5:

A jet plane traveling at 570 mph overtakes a propeller-driven plane that has a 2.8 hour head start. The propeller-driven plane is traveling at a rate of 150 mph. How far from the starting point does the jet overtake the propeller-driven plane?

Solution:

This time we have a problem involving moving objects. So here we will use the formula $r \cdot t = d$. Here, the two moving objects are the jet plane, and the prop plane. Also, we have been given the rate of each plane. So our chart starts like

	Rate	Time	Distance
Jet plane	570		
Prop plane	150		

In this case, we would like to know the distance, however, we will not be able to simply assign our distances as our variables. It turns out, in this case, that it is best to use put our variables in for time. The reason for this is that we have more information about the time then we do about the distance. So plugging in variables we can complete the chart.

	Rate	Time	Distance
Jet plane	570	x	570x
Prop plane	150	y	150y

Now, to get our system, we notice two things given in the problem that we have not yet used. The first is that the prop plane had a two hour head start. This means that it has been flying for 2.8 hours more than the jet plane. This gives us the equation y = x + 2.8.

Also, we are told that at some point, the jet overtakes the prop plane. So when the jet overtakes the prop plane, the distance that they traveled must be the same. So this gives us the equation 570x = 150y.

So our system is

$$\begin{aligned} y &= x + 2.8 \\ 570x &= 150y \end{aligned}$$

This system is obviously set up for solving with the substitution method. We only have to substitute the top equation into the bottom and solve.

$$\begin{aligned} 570x &= 150y \\ 570x &= 150(x + 2.8) \\ 570x &= 150x + 420 \\ -150x & \quad -150x \\ \frac{420x}{420} &= \frac{420}{420} \\ x &= 1 \end{aligned}$$

So the jet overtakes the prop plane when x = 1. This means, after the jet has been in the air for 1 hour, it passes the prop plane. However, the question was asking how far from the start does this occur. To figure this out, we just need to plug the x = 1 back into the chart for the distance. Therefore we get the distance of 570(1) = 570 miles.

The jet plane passes the prop plane after 570 miles.

Example 6:

A boat travels 36 miles in 4 hours upstream. In the same amount of time the boat can travel 48 miles downstream. Find the rate of the current and the rate of the boat in still water.

Solution:

Again here we are working with a moving object. So we will again use the formula $r \cdot t = d$. As we did in chapter 5, we will separate our chart into upstream verses downstream, or more accurately, against the current verses with the current. Also, we have been given a great deal of information we can just put directly in, such as times and distances. This gives us

	Rate	Time	Distance
Upstream a/current		4	36
Downstream w/current		4	48

As we saw in chapter 5, when we saw these moving objects before, the rate of travel against the current would be the normal speed of the boat minus the speed of the current (the current works against the boat and slows it the boat down) and the speed with the current would be the normal speed plus the speed of the current (the current works with the boat and speeds the boat up).

Since we are looking for the speed of the boat, and the speed of the current, lets label those to be our variables. Let's say x = speed of the boat in still water and y = speed of the current. So now our chart is

	Rate	Time	Distance
Upstream a/current	x – y	4	36
Downstream w/current	x + y	4	48

Now, how do we get the equations? In this case, we use the formula $r \cdot t = d$. That is to say, the rate column times the time column must equal the distance column. So we get the equations

$$(x - y) \cdot 4 = 36$$
$$(x + y) \cdot 4 = 48$$

Distribute our 4's and clearly the system is set up for elimination method.

$$\begin{aligned} 4x - 4y &= 36 \\ \underline{4x + 4y} &\underline{= 48} \\ 8x \quad\quad &= 84 \\ \frac{8x}{8} &= \frac{84}{8} \\ x &= 10.5 \end{aligned}$$

Now find the y value

$$\begin{aligned} 4x + 4y &= 48 \\ 4(10.5) + 4y &= 48 \\ 42 + 4y &= 48 \\ -42 \quad\quad &\;\; -42 \\ \frac{4y}{4} &= \frac{6}{4} \\ y &= 1.5 \end{aligned}$$

So the boat in still water goes 10.5 miles per hour, and the current is 1.5 miles per hour.

7.4 Exercises

1. At the donut shop, Marty orders his usual, hot chocolates and apple fritters. Hot Chocolate costs $.45 each and apple fritters cost $.30 each. His order costs $3.90. If Marty orders a total of 11 items, how many apple fritters did Marty buy?

2. A store sells 45 shirts, one kind at $8.50 and the other at $9.75. In all, $398.75 was taken in. How many of each type were sold?

3. A store owner purchased 70 light bulbs. He bought some 75-watt bulbs at $.50 each and fluorescent bulbs at $2 each. The total cost was $95. How many fluorescent light bulbs did he buy?

4. Bass Pro Shop sells 2 fishing reels and 5 fishing rods for $270. One reel and rod usually cost $84. How much does each cost individually?

5. At the theatre, Jon buys 2 boxes of popcorn and 3 soft drinks for $6.05. One box of popcorn and one soft drink would cost $2.60. What is the cost of a box of popcorn?

6. Marcus mixes two types of coffee beans, one that is $2 a pound and one that is $4 a pound. He wants to get 60 pounds of beans that cost $3 per pound. How much should Marcus mix of each type?

7. Jennifer has only nickels and quarters in her piggy bank. There are 34 coins totaling $4.30 in her piggy bank. How many of each type of coin does Jennifer have?

8. David has $2 worth of dimes and nickels in his hand. He has a total of 25 coins. How many of each does David have?

9. George bought Snickers bars and Licorice Whips for his class of 27 students as a reward for good behavior. Snickers bars cost him $0.40 each and Licorice whips cost him $0.30 each. He spent $9.60. How many of each did George buy?

10. Emma gives tennis lesson. She charges $100 an hour for a clinic and $50 an hour for a private lesson. If she worked for 40 hours this week and earned $3500, how many hours did she spend in private lessons?

11. After winning a tournament, a basketball team orders 4 pizzas and 9 sodas for their celebration. The bill came to $70.50. One pizza and a soda usually cost $14.50. How much does a single pizza cost?

12. Garth goes to the store to get change for a $50. He asks for some $5 bills and some $1 bills. As he walks out he finds that there are 22 bills in all. How many $5 does Garth now have?

13. A student is buying supplies for school. She buys 4 boxes of pencils and 7 reams of paper. The total cost is $13.25. Two months later, she makes a second trip for more supplies. This time she buys 3 boxes of pencils and 5 reams of paper at a total cost of $9.75. If the price is the same on both trips, what is the cost of a single box of pencils and a single ream of paper?

14. A store buyer purchased 12 regular calculators and 5 graphing calculators for a total cost of $370.75. A second purchase, at the same prices, included 10 regular calculators and 3 graphing calculators for a total of $246.25. Find the cost for each type of calculator.

15. On a history exam, a professor gives 3 points for each “true/false” choice question and 5 points for each “short answer” question. A student gets 20 questions correct and received 86 points. How many of each question type did the student answer correctly?

16. In Hockey, a team gets 2 points for each win and 1 point for each tie. The Kings won the Stanley Cup with a mere 95 points and had only 25 more wins than ties. How many wins and ties did the Kings have?

17. A field goal kicker has lost track of his stats this season. He knows that he only scores 3 points for a field goal and 1 point for a “point after”. If he also knows that he has made 47 kicks and has a total of 103 points, how many of each score did he make this season?

18. The COS Giant basketball team made 40 baskets in a recent game. Some were 2 point shots and the rest were 3 point shots. They ended the game with 89 points. How many of each type of basket did they make?

19. In the Modified Stableford system of scoring a round of golf, a birdie is worth 2 points, a par is worth zero (and so doesn’t affect the score) and bogey is worth -1 point. Mark ended a round of golf (18 holes) having scored 9 pars and the rest were birdies and bogeys. His score was a 3. How many birdies and bogeys did Mark score?

20. Lucky Gas made five hundred gallons of 89 octane gasoline by mixing 87 octane with 92 octane. How much of each did they use?

21. A metallurgist wishes to make 1800 lbs. of a 12% tin alloy by mixing a 20% tin alloy with an 8% tin alloy. How many pounds of each alloy are necessary?

22. Ten liters of 30% acid is made by mixing a 20% solution with a 50% solution. How much of each has been mixed?

23. A druggist has two solutions, one 60% hydrogen peroxide and the other 30% hydrogen peroxide. How many liters of each should be mixed to obtain 30 liters of a solution that is 40% hydrogen peroxide?

24. Tracy wants to mix an 8% alcohol solution with a 15% alcohol solution to get 100 ounces of a 12.2% alcohol solution. How much should she mix to get her desired results?

25. A researcher wishes to make 25 grams of a 75% aluminum alloy by mixing a 70% alloy with a 90% alloy. How many grams of each aluminum alloy are necessary?

26. Eric uses a cleaner that is 25% acid and cleaner that is 50% acid. How much of each should Eric mix if he wants 10 L of a cleaner that is 40% acid?

27. A chemist wants to produce 90 mL of a 42% acid solution by mixing a 40% acid solution with a 58% acid solution. How much of each solution should he mix?

28. Barbara sells nuts. She mixes a container holding 10% cashews with a container holding 40% cashews and gets 10 pounds of a mixture that contains 25% cashews. How much was in each starting container?

29. Frank mixes rum (40% alcohol) with Coke (no alcohol). He ends up with a drink that is 4 cups of 22% alcohol. How much of each did Frank mix?

30. A chemist has some 8% hydrogen peroxide solution and some 3% hydrogen peroxide solution. How many milliliters of the 8% solution should be used to make 500 milliliters of solution that is 4.2% hydrogen peroxide?

31. Andy wants to fertilize his 10 acres of peaches with 500 pounds of a 30% nitrogen fertilizer. It turns out he only has 10% nitrogen and 60% nitrogen available. How many pounds of each must he use to get the fertilizer that he wants?

32. John has two stains for painting his deck. One is 20% brown and the other is 60% brown. He needs 10 gallons to stain his deck. If John wants to have 45% brown, how much of each should he mix?

33. Donal has an antifreeze solution that is 25% antifreeze and a solution that is 77% antifreeze. He wants to mix these and get 100 ounces of a solution that is 38% antifreeze. How much of each solution must he use?

34. Professor Quigley mixes a pure acetone (100% acetone) with water (0% acetone) to get a 10% acetone mixture. If he produced 2 gallons of this mixture, how much of each did he mix?

35. How many grams of 10% pure gold must be mixed with 20% pure gold to make 25 grams of 16% pure gold?

36. How many grams of pure acid must be added to 20% acid to make 96 grams of solution that is 50% acid?

37. Dan has two types of feed for his cattle, one that is 12% protein and one that is 20% protein. How many pounds of each does Dan need to mix if he wants 50 pounds of feed that is 17% protein?

38. Don has two fertilizers for his yard. One contains 5% gypsum and the other contains 15% gypsum. How much of each should Don mix to get 100 pounds of a 12% gypsum fertilizer?

39. A chemist wishes to make 2000 liters of a 3.5% acid solution by mixing a 2% solution with a 5.5% solution. How many liters of each solution are necessary?

40. Jeff mixes 6% salt water with 3% salt water to get 2000 mL of a 5% salt water solution. How much of each did Jeff mix?

41. Shawn is training for a triathlon. He starts his workout by riding his bicycle for 1 hour and then he switches to running, which he does for an hour and a half. Shawn rides his bicycle 10 miles per hour faster than he can run. If he traveled a total of 27.5 miles, how fast does Shawn ride his bicycle?

42. A car drives 50 miles in the same amount of time that a plane fly's 180 miles. The car drives 143 miles per hour less than the plane. What is the speed of the plane?

43. A long haul trucker starts her trip by driving 65 miles per hour. When she reaches some road construction she is forced to slow her speed to 50 miles per hour. If it takes her 9 hours to travel 540 miles, how long was she stuck in the construction zone?

44. A train leaves Visalia at 9 am traveling to the northern bay area, a distance of 216 miles. One hour later a train leaves the northern bay area heading to Visalia. They meet at noon. If the second train left at 9 am and the first train left at 10:30 am, they still would have met at noon. What is the speed of each train?

45. Jon and Jen are taking a trip to Los Angeles. It is a 2.5 hour, 180 mile drive. Jon drives for a while but gets sleepy, so Jen takes over the drive. Jon drives 70 miles per hour while Jen drives 75 miles per hour. Who spent more time driving?

46. Jared takes a road trip. On the first day of his trip he drives 72 miles per hour. On the second day he travels 80 miles per hour. His total driving time over the two days is 15 hours and he covered a distance of 1148 miles. How far did Jared drive on the first day?

47. Shawn is working on his swimming for an upcoming triathlon. To do this he swims in a nearby river. Shawn swims for an hour downstream and covers 5 miles. Then he turns around and swims for an hour upstream and covers 3 miles. Find Shawn's swimming speed and the speed of the river.

48. A student pilot flies to a city at an average speed of 100 mph and then returns at an average speed of 150 mph. Find the total distance between the two cities if the total flying time was 5 hours.

49. A radio controlled airplane flying with the wind can travel 24 miles in 2 hours. Flying against the wind, the airplane can travel the same distance in 3 hours. Find the speed of the airplane in calm air and the speed of the wind.

50. Brad's boat took 3 hours to make a 90 mile trip downstream and 5 hours to make the same trip upstream. What is the speed of Brad's boat in still water?

51. Motorboat traveling with the current went 80 mi in 4 h. Against the current, the boat took 5 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.

52. Jason paddled his canoe for 4 hours with the current to reach his campsite 80 kilometers away. After his time camping, it took him 10 hours to make the return trip, against the current. What is Jason's paddling speed and what is the speed of the current?

53. A cabin cruiser traveling with the current went 60 mi in 3 h. Against the current, it took 5 hours to travel the same distance. Find the rate of the cruiser in calm water and the rate of the current.

54. With a tail wind a small plane can fly 290 miles in 2 hours. With the same head wind it can only fly 190 miles in the same amount of time. What is the speed of the plane and the speed of the wind?

55. A motorboat traveling with the current went 112 km in 4 h. Against the current, the boat could go only 80 km in the same amount of time. Find the rate of the boat in calm water and the rate of the current.

56. Flying with the wind, a plane flew 1000 miles in 4 hours. Flying against the wind, the plane could fly only 500 miles in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.

57. A jet can travel 1080 miles with the wind in 3 hours. Against the wind, the jet can only fly 960 miles in the same amount of time. What is the speed of the jet with no wind?

58. A plane traveling with the wind flew 3625 miles in 6.25 hours. Against the wind, the plane required 7.25 hours to go the same distance. Find the rate of the plane in calm air and the rate of the wind.

59. A small plane flying with the wind can travel 340 miles in 2 hours. Flying against the wind, the same plane can travel only 330 miles in 3 hours. Find the speed of the plane with no wind and the speed of the wind.

60. A rowing team rowing with the current traveled 45 mi in 3 h. Against the current, the team rowed 27 mi in 3 h. Find the rate of the rowing team in calm water and the rate of the current.