7.1 Solving Systems of Equations by Graphing

In this chapter we want to take a look at what happens when we put two linear equations, which we talked about in chapter 6, into a single problem. This gives us something called a system of equations.

<u>Definition</u>: **<u>System of Linear Equations</u>**- Two or more linear equations involving the same variables.

$$\begin{aligned} x - 3y &= -1 \\ 3x + y &= -2 \end{aligned} \qquad\qquad \begin{aligned} 3x - y &= -10 \\ 3x + y &= 4 \end{aligned}$$

Solutions to systems are ordered pairs that satisfy every equation in the system.

The first thing that we need to do is get a grip on the definitions.

<u>Example 1</u>:

a. Is (-1, 0) a solution to $\begin{matrix} x - 3y = -1 \\ 3x + y = -2 \end{matrix}$? b. Is (-1, 7) a solution to $\begin{matrix} 3x - y = -10 \\ 3x + y = 4 \end{matrix}$?

Solution:

a. To determine if the ordered pair is a solution, we need to determine if it satisfies each of the equations in the system. To do that, we simply plug the ordered pair in and see if we get a true statement.

$$\begin{aligned} x - 3y &= -1 \\ (-1) - 3(0) &= -1 \\ -1 &= -1 \end{aligned} \qquad\qquad \begin{aligned} 3x + y &= -2 \\ 3(-1) + (0) &= -2 \\ -3 &\neq -2 \end{aligned}$$

So even though the values works in the first equation, it does not work in the second equation. To be a solution, the ordered pair must work in **both**. So, (-1, 0) is not a solution to the system.

b. Again, we need to plug the ordered pair in and see what we get.

$$\begin{aligned} 3x - y &= -10 \\ 3(-1) - 7 &= -10 \\ -3 - 7 &= -10 \\ -10 &= -10 \end{aligned} \qquad\qquad \begin{aligned} 3x + y &= 4 \\ 3(-1) + (7) &= 4 \\ -3 + 7 &= 4 \\ 4 &= 4 \end{aligned}$$

This time, the ordered pair satisfies both equations, so (-1, 7) is a solution to the system.

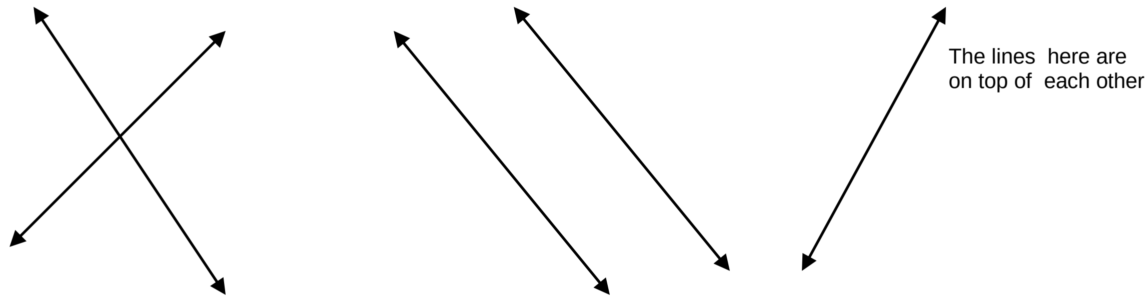
So far in this course we have seen a variety of different kinds of equations, and several different possibilities for the number of solutions that these equations could have.

This brings up the question, how many solutions can a linear system have?

Geometrically, since a solution to a system is any ordered pair that satisfies the equations, and since any ordered pair that satisfies an equation means that the point which the ordered pair represents is on the corresponding line, it follows that the solution to a system points of intersection of the graphs of the equations.

Therefore, in asking how many solutions a linear system can have, we are really asking…

How many ways can two lines intersects?



One intersection=
One solution

No intersections=
No Solution

Infinite intersections=
Infinite solutions

Now that we know what the possibilities are, we want to be able to solve systems.

It turns out there are numerous ways to solve a linear system. We will look at three different techniques in this chapter. The first technique comes right out of what we were just studying in the last chapter.

Solving a Linear System by Graphing

1. Graph each equation on the same set of axis.
2. The graphs intersect in:
    a. One point- one solution (called consistent)
    b. No points- no solution (called inconsistent)
    c. All points- infinite solutions (called dependent)
3. If the graphs intersect at one point, estimate the coordinates.

Even though this method seems reasonable, it turns out to be not as accurate as the other methods simply because we have to estimate the answer.

Nevertheless, solving by graphing is a good way to start the discussion of solving systems.

Example 2:

Solve the systems by graphing.

a. $\begin{array}{c} x+2y=4 \\ x+4y=10 \end{array}$ b. $\begin{array}{c} x-y=3 \\ 2x+3y=11 \end{array}$ c. $\begin{array}{c} 3x+y=10 \\ 6x+2y=5 \end{array}$

Solution:

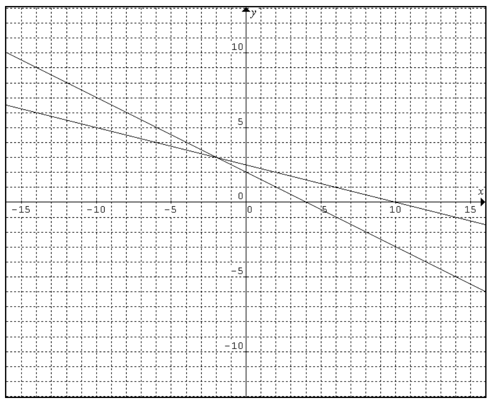
a. To solve the system by graphing, the first thing we need to do is to graph each equation. As we saw in chapter 6, the best way to do this is with the "slope-intercept-intercept" method. We, therefore, begin by putting each equation into slope intercept form.

$$\begin{aligned} x+2y&=4 \\ -x \quad &\;\; -x \\ \frac{2y}{2}&=\frac{-x}{2}+\frac{4}{2} \\ y&=-\frac{1}{2}x+2 \end{aligned} \qquad\qquad \begin{aligned} x+4y&=10 \\ -x \quad &\;\; -x \\ \frac{4y}{4}&=\frac{-x}{4}+\frac{10}{4} \\ y&=-\frac{1}{4}x+\frac{5}{2} \end{aligned}$$

So we see that the first line has a slope of – ½ and y-intercept of (0, 2) and the second line has a slope of – ¼ and a y-intercept of 2 ½ . Now we need to find the x-intercepts to use as our check points. As usual we set y = 0 and solve for x.

$$x + 2y = 4 \qquad\qquad x + 4y = 10$$
$$x + 2(0) = 4 \qquad\qquad x + 4(0) = 10$$
$$x = 4 \qquad\qquad x = 10$$

So our x-intercepts are (4, 0) for the first line, and (10, 0) for the second. Now we graph the lines on the same coordinate axis'.



Clearly we have one intersection point at (-2, 3). So the solution to the system is (-2, 3).

b. Just like in part a, we need to start by graphing our lines. Again, we will get the equations into slope-intercept form.

$$x - y = 3 \qquad\qquad 2x + 3y = 11$$
$$-x \quad\; -x \qquad\qquad -2x \quad\; -2x$$
$$\frac{-y}{-1} = \frac{-x}{-1} + \frac{3}{-1} \qquad\qquad \frac{3y}{3} = \frac{-2x}{3} + \frac{11}{3}$$
$$y = x - 3 \qquad\qquad y = -\frac{2}{3}x + \frac{11}{3}$$

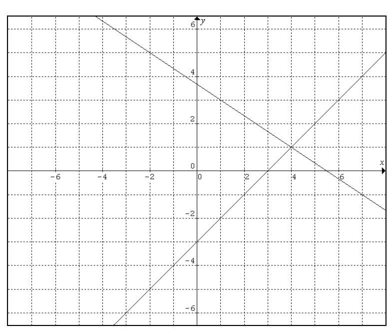
m = 1, y-int: (0, -3) m = $-\frac{2}{3}$, y-int: (0, $3\frac{2}{3}$)

Then we will get out x-intercept.

$$x - 0 = 3 \qquad\qquad 2x + 3(0) = 11$$
$$x = 3 \qquad\qquad 2x = 11$$
$$x = \frac{11}{2}$$

(3, 0) ($5\frac{1}{2}$, 0)

So graphing gives us



Therefore our system has a solution of (4, 1).

c. Lastly we graph each equation, by the same process we used in parts a and b. Start by getting the equations into slope-intercept form.

$$\begin{aligned} 3x + y &= 10 \\ -3x \quad &\quad - 3x \\ y &= -3x + 10 \end{aligned}$$

m = -3, y-int: (0, 10)

$$\begin{aligned} 6x + 2y &= 5 \\ -6x \quad &\quad - 6x \\ \frac{2y}{2} &= -\frac{6x}{2} + \frac{5}{2} \\ y &= -3x + \frac{5}{2} \end{aligned}$$

m = -3, y-int: (0, 2 ½)

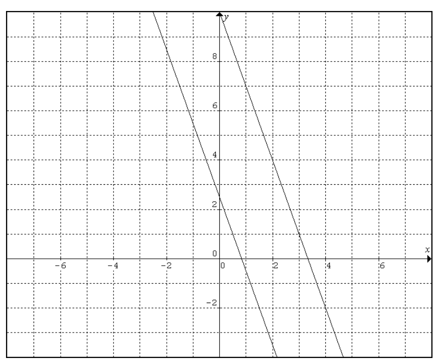
Notice that the lines have the same slope. We should recall from chapter 6 that when two lines have the same slope, they must be parallel. So in this case, the lines could be parallel or they could be the same line.

However, since they have different y-intercepts, they can't be the same line, ie they must be parallel. Let's finish our graphing to be sure. Find the x-intercepts for our check points.

$$\begin{aligned} 3x + y &= 10 \\ 3x + 0 &= 10 \\ \frac{3x}{3} &= \frac{10}{3} \\ x &= \frac{10}{3} \\ &\left(3\tfrac{1}{3}, 0\right) \end{aligned}$$

$$\begin{aligned} 6x + 2y &= 5 \\ 6x + 2(0) &= 5 \\ \frac{6x}{6} &= \frac{5}{6} \\ x &= \frac{5}{6} \\ &\left(\tfrac{5}{6}, 0\right) \end{aligned}$$

Graphing gives



Since the lines are clearly parallel, our system has no solution.

The case where a system has infinite solutions happens in a similar situation to Example 2 part c. above. The only difference is the equations would have the same slope and same x- and y-intercepts.

7.1 Exercises

Determine if the given ordered pair is a solution to the system.

1. $\begin{array}{c} y = x + 1 \\ x + y = -1 \end{array}$, (-4, -5)

2. $\begin{array}{c} 3x - y = 6 \\ x + 3y = 2 \end{array}$, (2, 0)

3. $\begin{array}{c} y = 2x + 11 \\ y - 5x = -19 \end{array}$, (-4, 3)

4. $\begin{array}{c} y = x - 5 \\ y + 2x = 4 \end{array}$, (-1, 3)

5. $\begin{array}{c} x + y = 1 \\ 3x - y = -5 \end{array}$, (-1, 2)

6. $\begin{array}{c} 3x + 2y = 6 \\ 2y = -6 \end{array}$, (1, 3)

7. $\begin{array}{c} y = 3x \\ 4y - 12x = 8 \end{array}$, (0, 0)

8. $\begin{array}{c} y = -6x \\ y = x - 5 \end{array}$, (0, 5)

9. $\begin{array}{c} y = x - 3 \\ 3x + 2y = 9 \end{array}$, (3, 0)

10. $\begin{array}{c} 2x - 5y = 1 \\ 4x + 3y = 0 \end{array}$, (3, 1)

11. $\begin{array}{c} 5x - 2y = 3 \\ x = y - 4 \end{array}$, $(\frac{11}{3}, \frac{23}{3})$

12. $\begin{array}{c} 8x - 2y = -5 \\ 3x + 4y = 1 \end{array}$, $(-\frac{9}{19}, \frac{23}{38})$

Solve the systems by graphing.

13. $\begin{array}{c} y = 6x - 11 \\ 2x + 3y = 7 \end{array}$

14. $\begin{array}{c} 7x + 2y = -13 \\ x - 2y = 11 \end{array}$

15. $\begin{array}{c} 2x - 3y = -1 \\ y = x - 1 \end{array}$

16. $\begin{array}{c} x + 2y = -4 \\ 4y = 3x + 12 \end{array}$

17. $\begin{array}{c} y = -3x + 5 \\ 5x - 4y = -3 \end{array}$

18. $\begin{array}{c} x + y = 0 \\ 3x + y = -4 \end{array}$

19. $\begin{array}{c} 3x + 3y = -3 \\ y = -5x - 17 \end{array}$

20. $\begin{array}{c} x - 3y = 6 \\ 2x - 6y = 6 \end{array}$

21. $\begin{array}{c} 6x - 4y = 2 \\ 3x - 2y = 4 \end{array}$

22. $\begin{array}{c} x + 3y = 3 \\ 2x - y = 6 \end{array}$

23. $\begin{array}{c} y = x - 1 \\ 2x - 2y = 2 \end{array}$

24. $\begin{array}{c} 6y = 2x - 12 \\ x - 3y = 6 \end{array}$

25. $\begin{array}{c} y = -2 \\ 4x - 3y = 18 \end{array}$

26. $\begin{array}{c} y = 1 \\ 2x + 3y = 3 \end{array}$

27. $\begin{array}{c} y = 5x - 7 \\ 3x + 2y = 12 \end{array}$

28. $\begin{array}{c} 3x + 8y = 20 \\ -5x + y = 19 \end{array}$

29. $\begin{array}{c} -5x + y = -2 \\ 3x - 6y = 12 \end{array}$

30. $\begin{array}{c} 2x + y = 1 \\ x - y = 5 \end{array}$

31. $\begin{array}{c} -4x + y = 6 \\ 5x + y = -21 \end{array}$

32. $\begin{array}{c} 2x - y = 3 \\ -4x + 2y = -6 \end{array}$

33. $\begin{array}{c} 5x - y = 3 \\ 3x - 8y = 24 \end{array}$

34. $\begin{array}{c} y = 2x \\ x + y = 3 \end{array}$

35. $\begin{array}{c} x + 3y = 1 \\ 3x + 3y = 15 \end{array}$

36. $\begin{array}{c} 3x + 4y = -2 \\ 3x + 3y = -3 \end{array}$

37. $\begin{array}{c} 3x - 3y = 4 \\ x - y = -3 \end{array}$

38. $\begin{array}{c} y = x + 2 \\ 4x + 3y = -15 \end{array}$

39. $\begin{array}{c} 6x + 6y = -6 \\ 5x + y = -13 \end{array}$

40. $\begin{array}{c} 2x + y = 9 \\ 5x - 2y = 18 \end{array}$

41. $\begin{array}{c} y = -2x + 1 \\ 6x + 3y = 3 \end{array}$

42. $\begin{array}{c} 3y = 2x - 3 \\ y = -x + 4 \end{array}$

43. $\begin{array}{c} x = 2 \\ 2x + 3y = 4 \end{array}$

44. $\begin{array}{c} 2y = 6x - 4 \\ 3x - y = 7 \end{array}$

45. $\begin{array}{c} y = x - 3 \\ 2x - 2y = 7 \end{array}$