6.4 Linear Inequalities

Now that we can graph linear equations, it only makes sense to change the equal sign to an inequality symbol and graph that as well.

<u>Definition</u>: **<u>Linear Inequality</u>**- A linear equation with an inequality symbol instead of an equal sign.

As it turns out, a solution to a linear inequality works the same as a solution to an equation, that is to say, an ordered pair is a solution if it satisfies the inequality.

<u>Example 1</u>:

Determine if the given point is a solution to the given inequality.

a. (-1, 3); $5x - 3y > 1$ b. (0, 1); $3x - 2y \leq 5$

Solution:

a. To see if the ordered pair is a solution, we need to find out if the ordered pair makes the inequality a true statement. So, all we need to do is plug it in and see what happens.

$$\begin{aligned} 5x - 3y &> 1 \\ 5(-1) - 3(3) &> 1 \\ -5 - 9 &> 1 \\ -14 &> 1 \end{aligned}$$

Since this is not a true statement, we can say (-1, 3) is not a solution to $5x - 3y > 1$

b. Similar to part a, we need only to plug the ordered pair in and see what we get

$$\begin{aligned} 3x - 2y &\leq 5 \\ 3(0) - 2(1) &\leq 5 \\ -2 &\leq 5 \end{aligned}$$

This time we have a true statement, so (0, 1) is a solution to $3x - 2y \leq 5$

Of course, just like with equations, there are an infinite number of possible solutions to an inequality. The main difference here is that with an inequality, we don't just have the ordered pairs that fall on the line, we have an entire half of the rectangular coordinate system as solutions.

So the only way that we can capture all the solutions is to graph it. We have the following method to do so.

<u>Graphing a Linear Inequality</u>
1. Replace the inequality symbol with an equal sign 2. Graph the equation (Dotted for $<$ and $>$; Solid for $\leq$ and $\geq$) 3. Test a point not on the line. If the point satisfies the inequality, shade the portion containing the point, if not, shade the other portion.

The reason for the dotted line for $<$ and $>$ verses the solid line for $\leq$ and $\geq$ is because with the $<$ or $>$ symbol, the equation isn't an included part of the solutions. So, the dotted line simply shows a guide as to where the line would be. However, for the $\leq$ and $\geq$, the equation is actually part of the solution since they have an “or equal to” component. So the solid line signals that the line itself is included in the solutions.

Example 2:

Graph the given inequality.

a. $x + y < 3$ b. $3x + 5y \geq 15$ c. $x - 3y < 6$

Solution:

a. To graph the inequality, the first thing we do is change $x + y < 3$ to $x + y = 3$. Now, we graph the equation the same way we always did, the “slope-intercept-intercept” method.

We start with putting the equation in the proper form

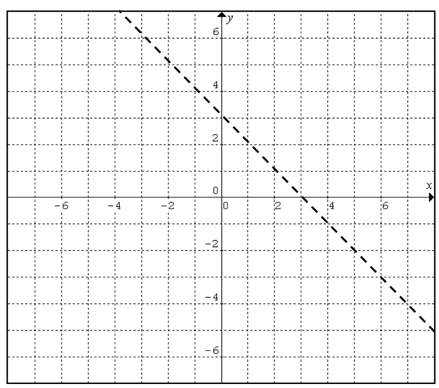
$$\begin{aligned} x + y &= 3 \\ -x \quad &\quad -x \\ y &= -x + 3 \end{aligned}$$

So m = -1 and y-intercept is (0, 3). Now we find the x-intercept by going back to the original equation and setting y = 0.

$$\begin{aligned} x + y &= 3 \\ x + 0 &= 3 \\ x &= 3 \end{aligned}$$

So the x-intercept is (3, 0).

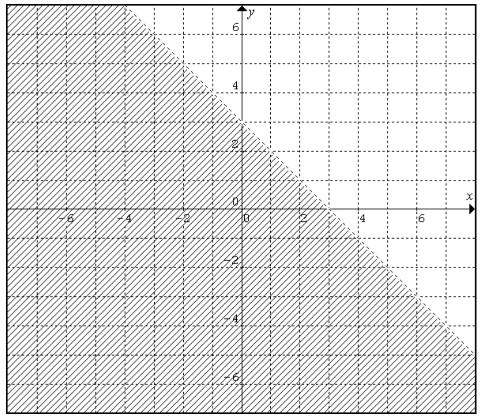
Now we draw the graph with a dotted line, since our inequality symbol is <.



Finally, to determine the proper shading, we need to test a point, any point, not on the line. To make it easy, lets test (0, 0). We do so by plugging it into the original inequality and seeing if we get a true statement.

$$\begin{aligned} x + y &< 3 \\ 0 + 0 &< 3 \\ 0 &< 3 \end{aligned}$$

So since we got a true statement, (0, 0) must be a solution. Therefore, every ordered pair in the region containing (0, 0) is a solution. So we shade the side of the line containing (0, 0) to get our completed graph.



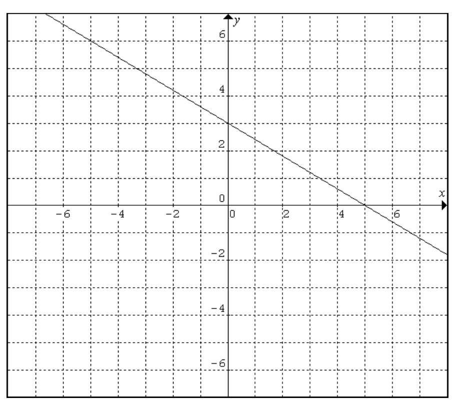
b. Again we start by changing $3x + 5y \geq 15$ to $3x + 5y = 15$. Then we get all the info we need to produce the graph.

$$3x + 5y = 15$$
$$-3x \qquad - 3x$$
$$\frac{5y}{5} = \frac{-3x}{5} + \frac{15}{5}$$
$$y = -\frac{3}{5}x + 3$$

So m = $\frac{-3}{5}$ and y-intercept is (0, 3).

$$3x + 5y = 15$$
$$3x + 5(0) = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

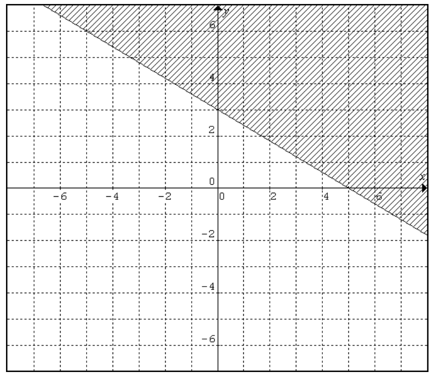
So our x-intercept is (5, 0). This time we graph with a solid line since we have a $\geq$.



Now lets again test the point (0, 0) for simplicity.

$$3x + 5y \geq 15$$
$$3(0) + 5(0) \geq 15$$
$$0 \geq 15$$

This is not a true statement. This means that (0, 0) cannot be in the shaded side which contains the solutions. So we have to shade the side that does not contain (0, 0) as follows



c. Finally, we again start by changing $x - 3y < 6$ to $x - 3y = 6$ and then graphing.

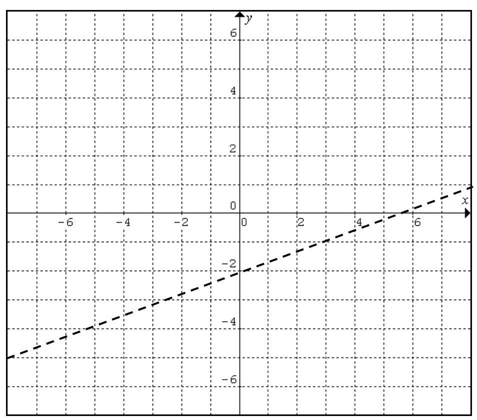
Slope-intercept:

$$x - 3y = 6$$
$$-x \qquad -x$$
$$\frac{-3y}{-3} = \frac{-x}{-3} + \frac{6}{-3}$$
$$y = \frac{1}{3}x - 2$$

x-intercept:

$$x - 3y = 6$$
$$x - 3(0) = 6$$
$$x = 6$$

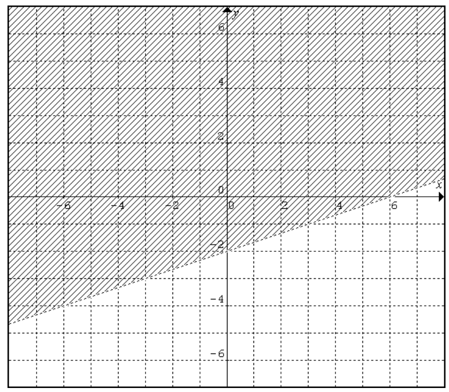
So m = $\frac{1}{3}$, y-intercept is (0, -2) and x-intercept is (6, 0). We graph with a dotted line because we are dealing with <.



Now we test (0, 0) to get

$$x - 3y < 6$$
$$0 - 3(0) < 6$$
$$0 < 6$$

So we shade the side containing (0, 0)



One cautionary note: Even though we used (0, 0) for every example here, you can actually use any point that you want, as long as it is not on the line. Since (0, 0) is very easy to use, we like to use it as often as possible. However, if (0, 0) turns out to be on the line, we would have to choose something else.

6.4 Exercises

Determine if the given point is a solution to the given inequality.

1. $(-2,3)$; $3x+5y<2$
2. $(1,2)$; $x-2y>0$
3. $(4,-2)$; $3x+2y\ge 6$
4. $(3,-1)$; $x-2y\le 4$
5. $(0,0)$; $2x+7y>1$
6. $(0,0)$; $2x-3y\ge 0$
7. $(-1,-4)$, $2x<2+y$
8. $(1,1)$; $4x-7<2y$
9. $(3,-1)$; $2x+y^2\ge 4$
10. $(0,-2)$; $4x^2-3y\le 2$

Graph the given inequality.

11. $y>-\frac{2}{3}x+1$
12. $y\le-\frac{3}{5}x-2$
13. $y\le -x-4$
14. $y\ge 2x-5$
15. $3x+2y<12$
16. $4x+5y>10$
17. $5x-y\le 3$
18. $2x-5y\ge 15$
19. $3x-4y>-8$
20. $3x>2-y$
21. $2x<3y-6$
22. $x-3y>3$
23. $x-4\ge 0$
24. $y+3\le 0$
25. $3x-5y<-10$
26. $x+y>2$
27. $y<\frac{2}{3}x$
28. $y\ge-\frac{1}{5}x$
29. $2x-5y>0$
30. $3x+y\le 0$