6.3 Slope of a Line

We saw in the last section that we can characterize a line by its intercepts. Another way we can characterize a line is by how steep the line is. That is to say, how much the line changes up and down, with respect to how much is changes left and right, as we travel along the line.

The idea of steepness of a line is called the slope of a line.

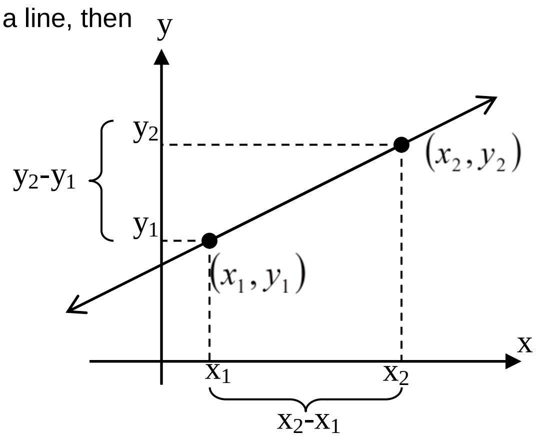
The idea of the slope will help us in a number of ways. First need a more rigorous definition of slope and a formula for slope as follows.

Definition: **Slope of a line**-

The slope of a line, m, is defined as follows:

Let (x_1, y_1) and (x_2, y_2) be two points on a line, then

$$m = \frac{change\ \ in\ \ the\ y\ direction}{change\ \ in\ \ the\ x\ direction} = \frac{y_2 - y_1}{x_2 - x_1}$$



First, we need to get a handle on the formula.

Example 1:

Find the slope of the line through the given points.

a. (2, 3), (5, 1) b. (4, 1), (-1, -2) c. (-4, 3), (4, 3) d. (-2, -1), (-2, 0)

Solution:

a. To find the slope of a line through a pair of given points, we need to use the formula given above. To do so, we have to label our points (x_1, y_1) and (x_2, y_2). The order of the points does not matter, however, we do need to make sure that each ordered pair has an "x" and a "y" and that the "1"'s are together and the "2"'s are together.

So, let's label (2, 3) as (x_1, y_1) and (5, 1) as (x_2, y_2). Then all we need to do is plug the values in and work through the formula as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{5-2} = \frac{-2}{3}$$

So the slope of the line through (2, 3) and (5, 1) is $m = -\frac{2}{3}$.

b. As is part a, we simply need to label our points (4, 1) as (x_1, y_1) and (-1, -2) as (x_2, y_2), then plug them into the slope formula as follows.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2-1}{-1-4} \\ &= \frac{-3}{-5} \\ &= \frac{3}{5} \end{aligned}$$

So this time the slope is $m = \frac{3}{5}$.

c. This time we label (-4, 3) as (x_1, y_1) and (4, 3) as (x_2, y_2). Plugging in gives

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-3}{4-(-4)} \\ &= \frac{0}{8} \\ &= 0 \end{aligned}$$

So, here m = 0.

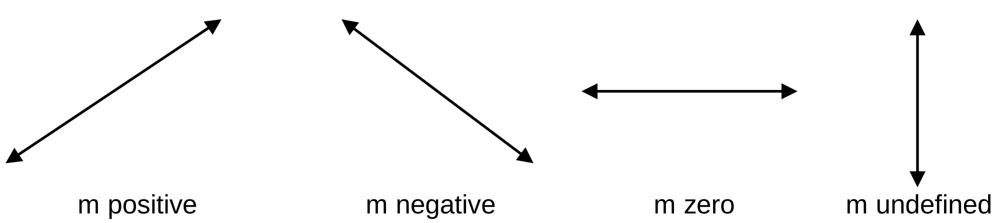
d. Lastly, we label (-2, -1) as (x_1, y_1) and (-2, 0) as (x_2, y_2). We get

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-(-1)}{-2-(-2)} \\ &= \frac{1}{0} \\ &= undefined \end{aligned}$$

So in this example, the slope is undefined. We will talk about what this means next.

We can see from Example 1 that slope can be positive, negative, zero or undefined. But what does this mean? Since slope is a highly geometric idea (steepness) we must be able to interpret these numbers geometrically.

The geometric interpretation for this is as follows:



Furthermore, the value of the slope determines the lines steepness. That is to say, the higher the value, the steeper the line. So a line with a slope of 5 would be much steeper than a line with a slope of 1, for example.

It's important to understand that the slope is a ratio of values that define how much a line changes in the y direction to how much a line changes in the x direction. Every point on the line, must maintain this ratio with every other point on the line.

Because of this idea, we can also, and more importantly use slope to produce the graph of a line. We use

<u>Slope-Intercept form of the equation of a line</u>
The graph of **y = *m*x + *b*** is a line whose slope is *m* and y-intercept is (0, *b*).

<u>Example 2</u>:

Graph by using the slope.

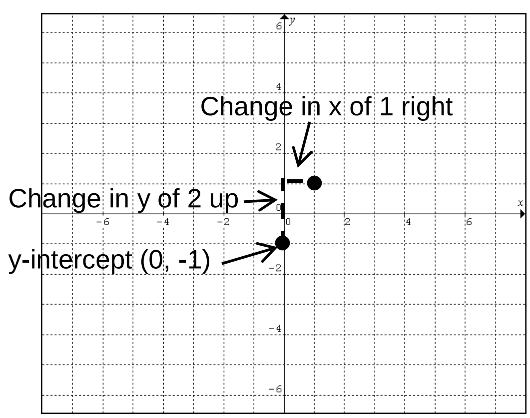
a. $y = 2x - 1$ b. $y = -\frac{1}{2}x + 2$

Solution:

a. First we should notice that the equation $y = 2x - 1$ is already in slope intercept form. Since that is the case, we can simply read the slope and y-intercept directly off of the equation. The slope is the coefficient of x, so m = 2 and the y-intercept is the constant term, so the y-intercept is (0, -1).

   Now, to use the slope and intercept to graph the line, we need to recall that the slope is defined as the change in the y direction over the change in the x direction. This means we should view the slope as $\frac{2}{1}$ instead of just 2. That way, we know that the change in the y direction (up and down) is 2 and the change in the x direction (left and right) is 1.

   So, we start by plotting the y-intercept, and then we move according to what the slope tells us, that is, 2 up (change in y, up/down is +2) and 1 right (change in x, left/right is +1) and put another point. We get the following



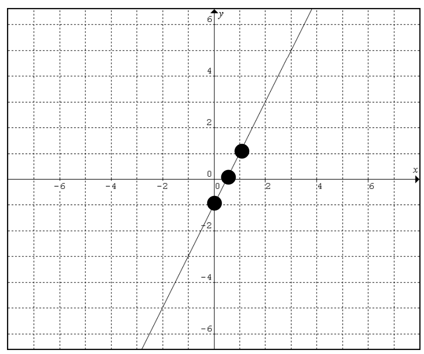
   So at this point, we have enough information to produce the graph, however, we always want to have at least three points in order to check our graph.

   There are a number of ways to get a third point. We could use a the slope to make another point, however, if we made a mistake from the first to the second point, we would make the same mistake from the second to the third point. So that would not serve as an adequate check point. We could also use a table of values to get a check point. But, the best check point to use is the x-intercept. The x-intercept is always a good piece of information to have, so using it as our check point would be a wise choice.

So we go back to our original equation to find our x-intercept. Set y = 0 as before

$$y = 2x - 1$$
$$0 = 2x - 1$$
$$+1 \qquad +1$$
$$\frac{1}{2} = \frac{2x}{2}$$
$$x = \frac{1}{2}$$

Plotting our x-intercept with our other points clearly checks our graph and so we simply need to draw the line to complete our graph.



b. Just like in part a. the equation is already in slope intercept form, y = - ½ x + 2. So, that means m = - ½ and the y-intercept is (0, 2).

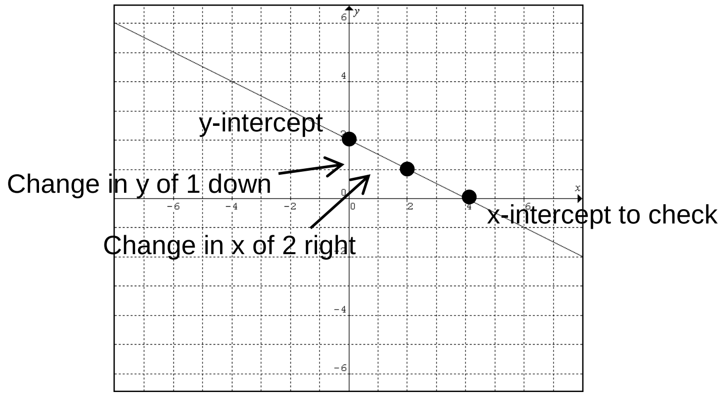
Also, just like above, we will need our x-intercept as our check point, so we will go ahead and calculate it now so that we can use it later.

$$0 = -\frac{1}{2}x + 2$$
$$-2 \qquad -2$$
$$-2 \cdot -2 = -\frac{1}{2}x \cdot -2$$
$$x = 4$$

So our x-intercept is (4, 0).

To graph, then, we start at the y-intercept and use the slope to generate another point. This time, since the slope is negative, its best to view it as $m = \frac{-1}{2}$. This way, the change in y is -1, which means down 1, and the change in x is +2, which means right 2. Whenever the slope is negative, its best to assign the negative to the numerator exclusively.

Putting this together we get.



Now that we have all of the pieces and parts we need, we can graph in general.

Clearly, the method show in Example 2 gives us the greatest amount of information and the best chance at getting the correct graph. For these reasons, this **"slope-intercept-intercept" method** for graphing is the preferred method for graphing lines.

<u>Example 3</u>:

Graph. Give the slope and the x-and y-intercepts.

a. 3x – 4y = 12   b. x + 3y = 6   c. x = -2   d. y = 1

Solution:

a. Since it provides all of the information we want anyway, we want to use the "slope-intercept-intercept" method to graph, but first we notice that the equation is not in slope intercept form. So we need to start solving for y so that the equation will be in slope intercept form.

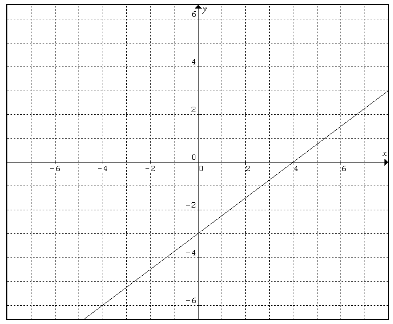
$$3x - 4y = 12$$
$$-3x \qquad -3x$$
$$\frac{-4y}{-4} = \frac{-3x + 12}{-4}$$
$$y = \frac{3}{4}x - 3$$

So $m = \frac{3}{4}$ and the y-intercept is (0, -3). Now let's calculate our x-intercept. Its best to go all the way back to the original equation so that the x-intercept will be a completely independent check point. Thereby ensuring that it truly does check our graph.

$$3x - 4(0) = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

So our x-intercept is (4, 0).

Starting with the y-intercept and the slope (remembering that m = ¾ means change in y of up 3 and change in x of right 4), then using our x-intercept to check we get



b. Again, let's start by getting the equation into slope intercept form.

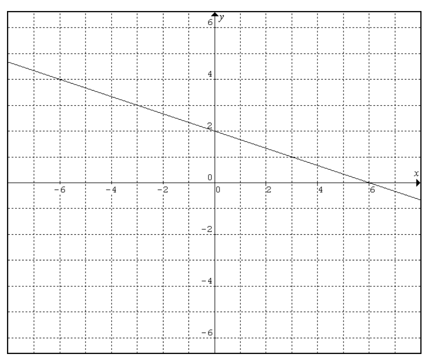
$$\begin{aligned} x + 3y &= 6 \\ -x \quad &\quad - x \\ \frac{3y}{3} &= \frac{-x}{3} + \frac{6}{3} \\ y &= -\frac{1}{3}x + 2 \end{aligned}$$

So $m = -\frac{1}{3} = \frac{-1}{3}$ and the y-intercept is (0, 2). Now calculating our x-intercept we have

$$\begin{aligned} x + 3(0) &= 6 \\ x &= 6 \end{aligned}$$

Our x-intercept is (6, 0).

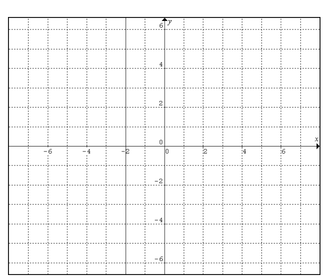
So, to graph we use plot the y-intercept, and use the slope (change in y of -1 means down 1, change in x of 3 means 3 right), then plot the x-intercept to check the graph.



c. To graph x = -2, we need to remember that in the last section we saw that every equation of the form x = a, is a vertical line through the point (a, 0) which is an x intercept, and vertical lines have no y intercept. Also, earlier in this section we mentioned that every vertical line has an undefined slope.

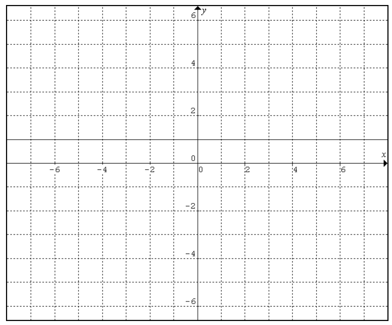
So putting this together we have x-intercept of (-2, 0), no y-intercept and m = undefined.

This gives us the graph



d. Lastly, we also saw in the last section that y = b is a horizontal line and in this section we said that every horizontal line has a 0 slope.

So y = 1 must give us no x-intercept, y-intercept of (0, 1) and m = 0. We get



6.3 Exercises

Find the slope of the line through the given points.

1. $(2,5)$ and $(3,9)$
2. $(5,11)$ and $(6,4)$
3. $(-2,3)$ and $(4,9)$
4. $(3,-5)$ and $(4,1)$
5. $(3,2)$ and $(-4,-7)$
6. $(-3,-4)$ and $(1,2)$
7. $(-2,-5)$ and $(3,-5)$
8. $(2,4)$ and $(-4,4)$
9. $(2,-7)$ and $(2,5)$
10. $(-3,1)$ and $(3,-1)$
11. $(5,-7)$ and $(-5,7)$
12. $(4,10)$ and $(2,-1)$
13. $\left(-\frac{1}{2},3\right)$ and $\left(\frac{3}{2},-1\right)$
14. $\left(\frac{1}{2},\frac{3}{4}\right)$ and $\left(\frac{1}{3},-\frac{1}{4}\right)$
15. $(0,32)$ and $(100,212)$
16. $(2,4)$ and (a,a^2)
17. $(1,1)$ and $\left(b,\frac{1}{b}\right)$
18. $(-2,-8)$ and (c,c^3)
19. (a,a^2) and $(a+h,(a+h)^2)$
20. $\left(b,\frac{1}{b}\right)$ and $\left(b+h,\frac{1}{b+h}\right)$

Find the slope of the line by putting the equation in slope-intercept form.

21. $3x+y=2$
22. $6x+y=1$
23. $\frac{1}{2}x+y=5$
24. $\frac{2}{3}x+y=7$
25. $-3x+2y=4$
26. $5x-3y=6$
27. $7x+9y=-18$
28. $3x-y=7$
29. $2x-3y=5x-1$
30. $3x-3=2y-4$
31. x = -3
32. y = 4

Graph. Give the slope and the x-and y-intercepts.

33. $y=3x+1$
34. $y=-2x-1$
35. $y=-\frac{1}{2}x+3$
36. $y=\frac{3}{4}x-2$
37. $y=2x$
38. $y=-3x$
39. $y=-\frac{6}{5}x$
40. $y=\frac{4}{3}x$
41. $y=5x-1$

42. $2x+3y=6$

43. $3x-5y=15$

44. $2x-y=-4$

45. $x+3y=-6$

46. $3x+4y=8$

47. $2x-5y=5$

48. $7x+2y=-4$

49. $5x-3y=9$

50. $x-y=4$

51. $3x+4y=-2$

52. y = 3

53. y = -1

54. x = -5

55. x = 6

56. $y=-\frac{3}{2}x-\frac{1}{2}$

57. $2x+y=3$

58. $3x-y=1$

59. $x-y=0$

60. $4x+2y=0$

61. $5x-2y=4$

62. $3x-4y=8$

63. $-2x-7y=14$

64. $\frac{1}{2}x+\frac{2}{3}y=\frac{4}{3}$

65. $\frac{3}{4}x-\frac{3}{8}y=\frac{3}{16}$

66. $2x+3y+4=3x-3y+4$

67. $3x+4y=5x+4y-1$