6.2 Graphing Lines by Intercepts

Let's start with graphing a couple more lines using a table of values. We will use these to make some observations.

<u>Example1</u>:

Graph.

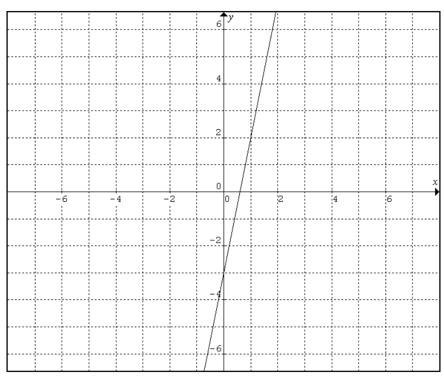
a. $y = 5x – 3$ b. $y = -2x + 4$

Solution:

a. We will use a table of values as we did in the previous section, however, we will only use three points, x = -1, 0, and 1, since three points will provide a sufficient amount of data to create the graph.

x	y	
-1	-8	$y = 5(-1) – 3 = -5 – 3 = -8$
0	-3	$y = 5(0) – 3 = 0 – 3 = -3$
1	2	$y = 5(1) – 3 = 5 – 3 = 2$

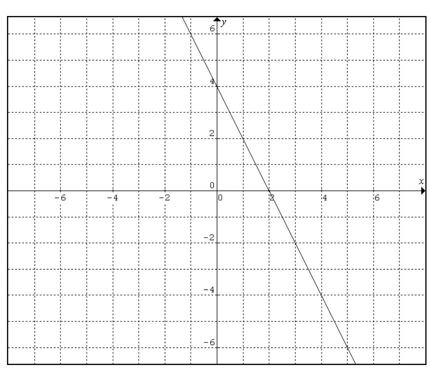
Plotting the points we get



b. We use the same process as above.

x	y	
-1	6	$y = -2(-1) + 4 = 2 + 4 = 6$
0	4	$y = -2(0) + 4 = 0 + 4 = 4$
1	2	$y = -2(1) + 4 = -2 + 4 = 2$

Plotting the points we get



Now, there are a number of things we can observe about the two graphs from Example 1. We will talk about them in the next couple of sections. First, notice that each graph crosses the x- and y-axis.

These are called the intercepts.

Definitions;
x-intercept- the point at which the graph hits the x-axis

y-intercept- the point at which the graph hits the y-axis

So how do we find these intercepts? It is actually quite easy. As we can see from the previous section and the example above, for a point to be on the x-axis, the y value must be zero, and similar for a point to be on the y-axis, the x value must be zero.

For this reason we have

Finding x- and y-intercepts

1. To find the x-intercept, Let y = 0 and solve for x. This is the point (x, 0).
2. To find the y-intercept, Let x = 0 and solve for y. This is the point (0, y).

It's easy to remember this. Just remember, whichever intercept you want to find, set the other variable equal to zero and solve.

Example 2:

Find the x- and y-intercepts and use them to graph.

a. y = x + 5 b. x – 3y = 6 c. x + y = 0

Solution:

a. The first thing that we need to do is to find the intercepts. Using the method above to do so is fairly straight forward. To get the x-intercept, we set y = 0 and to get the y-intercept, we set x = 0. We get the following

x-intercept:
$0 = x + 5$
$-5 \quad -5$
$x = -5$

y-intercept:
$y = 0 + 5$
$y = 5$

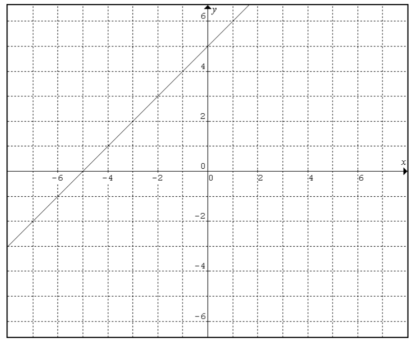
So we get (-5, 0) and (0, 5) for our intercepts.

Although any two points determines a line, we always like to have at least three points just to verify our graph and all of our other work. We call this third point the "check point". We simply need to make a table of values for one point. We can choose any value that we want, as long as it is not a value that we have already used. So how about x = 1. This gives

x	y
1	6

y = (1) + 5 = 6

Now we plot our intercepts and our check point to get our graph.



b. Again we start by finding the intercepts.

x-intercept:

$x - 3(0) = 6$

$x = 6$

y-intercept:

$0 - 3y = 6$

$-3y = 6$

$y = -2$

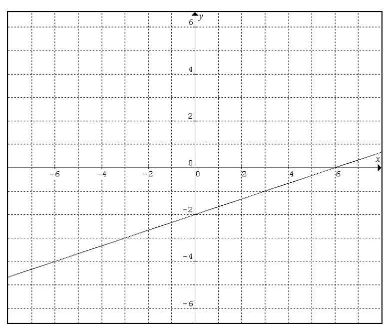
So we get (6, 0) and (0, -2) for our intercepts.

Now we find our "check point". This time let's use x = 3. The reason for this is, looking at the equation, 3 appears to be a value that might work out well, however, keep in mind that any value will work.

x	y
3	-1

$$3 - 3y = 6$$
$$-3 \quad\quad -3$$
$$\frac{-3y}{-3} = \frac{3}{-3}$$
$$y = -1$$

Plotting gives



c. Lastly, we find the intercepts as we did in parts a. and b.

x-intercept:

$x + 0 = 0$

$x = 0$

y-intercept:

$0 + y = 0$

$y = 0$

So we get (0, 0) and (0, 0) for our intercepts. This is the same exact point. In fact, with lines, when one intercept is 0, the other intercept must also be 0. So this time we will need to generate two "check points" because we always prefer to have at least three points to define our lines.

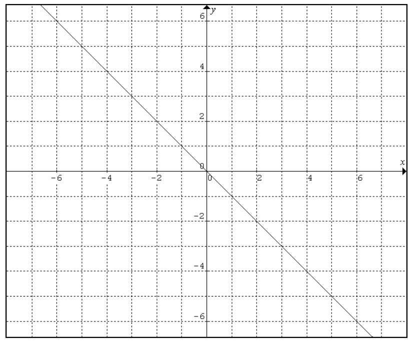
Let's use x = -1 and 1.

x	y
-1	1
1	-1

-1 + y = 0
+1 +1
y = 1

1 + y = 0
-1 -1
y = -1

So we get the following graph



Now that we can graph a basic line by using the intercepts, let's take a look at a couple of "glitches" to lines.

<u>Example 3</u>:

Graph by using the intercepts.

a. y = -2 b. x = 3

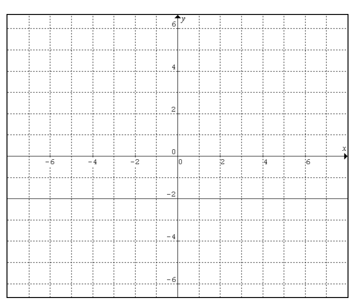
Solution:

a. We will begin, as we did in Example 2, by finding the intercepts.

<u>x-intercept</u>:
When we set y = 0
We get an unusual result
$0 = -2$
This must mean there is
No x-intercept

<u>y-intercept</u>:
Since the equation has no "x" when we set it equal to zero we get
$y = -2$

So we have a graph with no x-intercept, but a y-intercept of (0, -2). The only kind of line that can hit the y-axis at -2 but not ever touch the x-axis, would be a horizontal line through the point (0, -2). So our graph must be

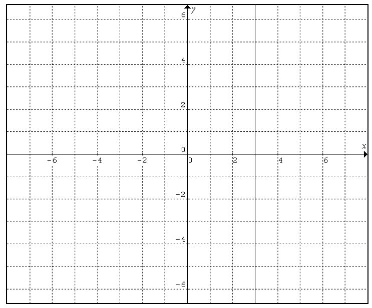


b. In the same way, it seems to be logical that the equation x = 3 would have to be a vertical line through the value (3, 0). We can verify this by finding our intercepts as below.

<u>x-intercept</u>:
$x = 3$

<u>y-intercept</u>:
$0 = 3$, No y-intercept

So our graph is



As it turns out, from example 3 we have the following

Horizontal lines and Vertical lines
All horizontal lines are of the form **y = b**, where **b** is a real number. All vertical lines are of the form **x = a**, where **a** is a real number.

This works either way. That is to say, a line of the form y = b is horizontal, and a horizontal line is of the form y = b. The same can be said for vertical lines. This will be important later in this chapter.

6.2 Exercises

Find the x and y-intercepts.

1. $2x+3y=6$
2. $3x-5y=15$
3. $2x-y=-4$
4. $x+3y=-6$
5. $3x+4y=8$
6. $2x-5y=5$
7. $7x+2y=-4$
8. $5x-3y=9$
9. $x-y=4$
10. $3x+4y=-2$
11. $y=-2$
12. $x=3$

Graph using the intercepts.

13. $2x+y=4$
14. $x+3y=6$
15. $x-y=0$
16. $2x+3y=6$
17. $4x-y=2$
18. $5x-4y=-20$
19. $x+7y=-14$
20. $-2x-y=9$
21. $1+2y=4x$
22. $3x=4-y$
23. $x=2y-8$
24. $4x=5y$
25. $y=2x-3$
26. $y=-3x+5$
27. $y=x+1$
28. $y=\frac{1}{2}x-2$
29. $2x+y=2$
30. $x+y=6$
31. $x+y=0$
32. $2x+3y=6$
33. $4x-y=8$
34. $5x-y=-20$
35. $2x+7y=-14$
36. $-2x-y=9$
37. $x=3$
38. $y=-1$
39. $y=-4$
40. $x=-2$
41. $y=5$
42. $y=0$
43. $x=0$
44. $x=2$
45. $y=-\frac{3}{4}x+2$
46. $y=-\frac{2}{3}x-3$
47. $y=\frac{3}{5}x+7$
48. $y=1\frac{2}{5}x+4$
49. $\frac{1}{2}x+\frac{1}{3}y=4$
50. $\frac{2}{3}x+\frac{1}{4}y=\frac{1}{2}$
51. $\frac{2}{7}x-\frac{1}{14}y=\frac{3}{4}$
52. $\frac{1}{6}x-\frac{2}{9}y=1$