5.8 Applications of Rational Expressions

The last thing we want to do with Rational Expressions is the last thing we always want to do when we learn a new topic. That is, we want to talk about applications to the real world.

In this case, we will only deal with two different types of word problems. They are problems that involve a moving object, and problems that involve work.

We need the following formulas:

For problems involving moving objects (cars, trains, people running, etc):

$$r \cdot t = d$$

Where r is the rate of speed of the moving object, t is the time the object moves, and d is the distance the object travels.

For problems involving people or machines doing work:

$$rate\ of\ work \cdot time\ worked = part\ of\ job\ completed$$

Or more simply,

$$R \cdot t = P.C.$$

Where, the rate of work is the amount of the job that gets done per unit of time. So it turns out to be $\frac{1}{time\ to\ do\ the\ job\ alone}$

In order to work these word problems, we will need to use chart to organize the information. The chart will help us to pull the information out of the problem and generate our equation.

In general, the charts look like this

	Rate	Time	Distance or Part Completed
Object 1			
Object 2			

The "Object 1" and "Object 2" in the table are usually fairly clear in the problem. For instance, on a work problem, they would likely be the two "objects" that are doing the work, and in a moving object problem, they would be either "objects" that move at two different rates, or two different moving objects.

On the top row of the table is always the formula that is used for whichever type of problem you are doing.

With all of this a background, let's take a look at a couple of each type of problem, beginning with work.

<u>Example 1</u>:

It takes Jordan 36 minutes to mow the lawn while it takes James 45 minutes to mow the same lawn. If Jordan and James work together, using two lawn mowers, how long would it take them to mow the lawn?

Solution:

First, we start by getting our table going. To start off, the first thing we should notice is that we are doing a work problem. So the formula for the top row is Rate x Time = Part Completed.

Also, since we have two different people doing the work, Jordan and James, these would be our "Object 1" and "Object 2". So, our chart initially looks like

	Rate	Time	Part Completed
Jordan			
James			

Now, we need to start filling the chart in. Usually, we want to only fill two of the rows and use the formula to fill in the rest.

Recall from earlier in the section we stated that the rate part of the chart is $\frac{1}{\textit{time to do the job alone}}$. Since it takes Jordan 36 minutes and James 45 minutes to mow the lawn alone, there rates must be $\frac{1}{36}$ and $\frac{1}{45}$, respectively.

So our chart now looks like

	Rate	Time	Part Completed
Jordan	$\frac{1}{36}$		
James	$\frac{1}{45}$		

Since we are looking to find the amount of time that it takes for them to do the job working together, we need to assign our variable to the time column. So this gives us

	Rate	Time	Part Completed
Jordan	$\frac{1}{36}$	x	
James	$\frac{1}{45}$	x	

Now we simply fill in the last column by using the formula, that is,

Rate x Time = Part Completed

So, the rate column times the time column must be the part completed column. That makes our chart look like

	Rate	Time	Part Completed
Jordan	$\frac{1}{36}$	x	$\frac{x}{36}$
James	$\frac{1}{45}$	x	$\frac{x}{45}$

Now that our chart is complete, we need to use it to generate our equation. The idea is very simple. The last column represents the amount of the job that each person, Jordan and James, completed.

That is to say, the part of the lawn that Jordan mowed is $\frac{x}{36}$ of the lawn, and the part that James mowed is $\frac{x}{45}$ of the lawn. Since we want them to mow the entire lawn, working together, we want the part that Jordan mows plus the part that James mows to add up to 1 entire lawn. Therefore, our equation must be

$$\frac{x}{36} + \frac{x}{45} = 1$$

Notice, the right hand side of the equation will always represent the portion of the job we want to be completed. So, if we only wanted half of the lawn mowed, it would have said ½ , if we wanted the lawn mowed twice, it would have said 2, etc.

In any case, now that we have our equation, we just need to solve like we always did.

$$\frac{x}{36} + \frac{x}{45} = 1$$

Multiply by the LCD of 180

$$180 \cdot \left(\frac{x}{36} + \frac{x}{45}\right) = (1) \cdot 180$$

Distribute the 180 through both sides

$$\frac{180x}{36} + \frac{180x}{45} = 180$$

Reduce

$$5x + 4x = 180$$

Combine like terms

$$9x = 180$$

Divide by 9 to get x alone

$$x = 20$$

So, it should take 20 minutes for Jordan and James to mow the lawn working together.

You can see that this type of example can prove to be very useful for a business owner who has employees. Say, for example, if Jordan and James were two employees of a landscaping company. If the owner sends them out on a job like the one in the example and they take more than 20 minutes to finish, Jordan and James would have some explaining to do.

Lets look at another work problem.

<u>Example 2</u>:

Working together, Jennifer and Lori can plant a vegetable garden in 3 hours. If Lori works alone, it takes her 8 hours longer than it takes Jennifer to plant the garden. How long does it take Lori to plant the vegetable garden by herself?

Solution:

Here again we have another work type of problem. So we will, again use the formula for work, Rate x Time = Part Completed. Also, notice that we have two people planting the garden, Jennifer and Lori. They will be our "Objects. So our chart starts as

	Rate	Time	Part Completed
Jennifer			
Lori			

In this case, we have been given the time that they work together, but not the time it takes them to do the job alone. Remember, the rate is based on the time to complete the job alone.

Since they didn't give us those times, we will have to use a variable. Since we are looking for Lori's time to complete the job alone, let's call her time x. Since it take Lori 8 hours longer than Jennifer, this would mean Jennifer's time to complete the job by herself would be x - 8 (since she is 8 hour faster than Lori).

So, since the rate is $\frac{1}{time\ to\ do\ the\ job\ alone}$, our chart must now look like

	Rate	Time	Part Completed
Lori	$\frac{1}{x}$	3	
Jennifer	$\frac{1}{x-8}$	3	

Now we multiply straight across to fill in the last column. This gives

	Rate	Time	Part Completed
Lori	$\frac{1}{x}$	3	$\frac{3}{x}$
Jennifer	$\frac{1}{x-8}$	3	$\frac{3}{x-8}$

As in example 1, the part completed for each is the portion of the planting the garden that they each did. Since we want them to only plant the garden once, the equation must be Lori's part + Jennifer's part = 1 vegetable garden. That is,

$$\frac{3}{x}+\frac{3}{x-8}=1$$

Now we solve as usual.

$$\frac{3}{x}+\frac{3}{x-8}=1$$

Multiply by the LCD

$$x(x-8)\cdot\left(\frac{3}{x}+\frac{3}{x-8}\right)=(1)\cdot x(x-8)$$

Distribute

$$\frac{3x(x-8)}{x}+\frac{3x(x-8)}{x-8}=x(x-8)$$

Reduce

$$3(x-8)+3x=x(x-8)$$

Multiply out what is left

$$3x-24+3x=x^2-8x$$

Combine like terms

$$\begin{aligned} 6x-24 &= x^2-8x \\ -6x+24 &\quad\ -6x+24 \end{aligned}$$

Move everything to one side

$$x^2-14x+24=0$$

We will have to solve with factoring since we have a variable that is squared.

$$x^2 - 14x + 24 = 0$$

$$(x - 12)(x - 2) = 0$$

$$\begin{array}{ll} x - 12 = 0 & x - 2 = 2 \\ \;\;+12 \quad +12 & \;\;+2 \quad +2 \\ \\ \quad\; x = 12 & \quad\; x = 2 \end{array}$$

Since we have two answers, we have to determine which one is correct. Whichever answer makes sense in the entire problem would be the correct answer. In this case, if Lori only took 2 hours to plant the garden, it would take Jennifer 8 hours less, or 2 - 8 = -6 hours. This answer is ridiculous. Therefore, the answer must be that it takes Lori 12 hours to plant the vegetable garden.

Now, let's look at a couple of problems involving moving objects.

Example 3:

Adam drives 15 miles per hour faster than David does. Adam can drive 100 miles in the same amount of time that David drives 80 miles. Find Adams driving speed.

Solution:

Since, this time, we are talking about a moving object, our formula is Rate x Time = Distance. Also, clearly we have two moving objects, Adam and David. Therefore, our chart starts as

	Rate	Time	Distance
Adam			
David			

Since we want to know Adams driving speed, lets assign Adams rate of speed to x. This would make David's driving speed x - 15 (David is slower than Adam). Also, clearly we have the distance that each has traveled given in the problem. Adam drives 100 miles, and David drives 80. So filling in, our chart looks like

	Rate	Time	Distance
Adam	x		100
David	x - 15		80

At this point, we notice that this is very different than the charts in examples 1 and 2. In those examples we simply needed to multiply across to fill in the chart. Here, however, we are missing a value in the middle of the chart.

This is where the formula comes in. We are missing the "time" part of the chart, so we can take the formula $r \cdot t = d$ and solve it for *t* like we did in the last section. We have

$$r \cdot t = d$$

$$\frac{r \cdot t}{r} = \frac{d}{r}$$

$$t = \frac{d}{r}$$

So, time equals distance over rate. We can use this to complete the table. We have

	Rate	Time	Distance
Adam	x	$\frac{100}{x}$	100
David	x - 15	$\frac{80}{x-15}$	80

Now, to get the equation, we have to go back to the problem and see how the times are related. Notice in the original problem the phrase "in the same amount of time" is used. This means, the time that Adam takes is the same (or equal) to the time that David takes. This must mean our equation is

$$\frac{100}{x} = \frac{80}{x-15}$$

Now we just solve

$$x(x-15)\cdot\left(\frac{100}{x}\right) = \left(\frac{80}{x-15}\right)\cdot x(x-15)$$

Multiply by the LCD

$$\frac{100x(x-15)}{x} = \frac{80x(x-15)}{x-15}$$

Reduce

$$100(x-15) = 80x$$

Multiply out the sides

$$\begin{aligned} 100x - 1500 &= 80x \\ -100x \qquad &\quad -100x \end{aligned}$$

Move 100x over

$$-20x = -1500$$

Divide by -20 to get x alone

$$\frac{-20x}{-20} = \frac{-1500}{-20}$$

$$x = 75$$

So, Adam is driving 75 miles per hour.

In our final example we will look at a tougher moving object problem.

<u>Example 4</u>:

Jon is kayaking in the Russian River which flows downstream at a rate of 1 mile per hour. He paddles 5 miles downstream and then turns around and paddles 6 miles upstream. The trip takes 3 hours. How fast can Jon paddle in still water?

Solution:

Here again we are doing a moving object problem, but this time our rows will need to be labeled as "with the current" (downstream) and "against the current" (upstream). So our chart starts as

	Rate	Time	Distance
With the current			
Against the current			

Just like in the example above, we want to fill in as much information as we can before using the formula to complete the chart.

Here we see that the distance traveled with the current is 5 miles and against the current is 6 miles. Those values are easy. The rate, on the other hand is more difficult.

First of all, since we are looking for the speed in "still water", this means we are looking for the speed when there is no current. Lets call that speed x.

Since our speed with no current is x and the speed of the current is 1 mile per hour, the rate of speed with the current must be x + 1. This is because if you are traveling with the current, you will speed up by exactly the speed of the current (it works with you). Likewise, the speed against the current would be x - 1, since the current works against you.

This gives us

	Rate	Time	Distance
With the current	x+1		5
Against the current	x-1		6

Now fill in the chart like we did on the last example, that is, time equals distance over rate.

	Rate	Time	Distance
With the current	x+1	$\frac{5}{x+1}$	5
Against the current	x-1	$\frac{6}{x-1}$	6

To get the equation, we look back at the original problem. Here is says that the total amount of time for the trip was 3 hours. That means the time going upstream plus the time going downstream took a total of 3 hours.

So the equation must be

$$\frac{5}{x+1}+\frac{6}{x-1}=3$$

Now we solve.

$$(x+1)(x-1)\cdot\left(\frac{5}{x+1}+\frac{6}{x-1}\right)=(3)\cdot(x+1)(x-1)$$ Multiply by the LCD

$$\frac{5(x+1)(x-1)}{x+1}+\frac{6(x+1)(x-1)}{x-1}=3(x+1)(x-1)$$ Distribute the LCD

Reduce

$$5(x-1)+6(x+1)=3(x+1)(x-1)$$

Multiply out each side

$$5x-5+6x+6=3(x^2-1)$$

$$\begin{array}{rl} 11x+1 &= 3x^2-3 \\ -11x-1 & \quad -11x-1 \end{array}$$ Move everything to one side

$$3x^2-11x-4=0$$

Factoring gives

$$3x^2 - 11x - 4 = 0$$

$$(3x + 1)(x - 4) = 0$$

$$\begin{array}{rcl} 3x + 1 = 0 & & x - 4 = 0 \\ -1 \quad -1 & & +4 \quad +4 \end{array}$$

$$3x = -1 \qquad x = 4$$

$$\frac{3x}{3} = \frac{-1}{3}$$

$$x = -\frac{1}{3}$$

Since x represents a rate of speed, speed can't be negative, the answer must be the 4. Therefore, Jon can paddle 4 miles per hour is still water.

This problem is very similar to another problem involving planes that fly with the wind and against the wind. The "against the wind/with the wind" problem is done just like the "upstream/downstream" problem

5.8 Exercises

1. One water pipe can fill a tank in 10 minutes, while a second pipe can fill the same tank in 15 minutes. How long would it take to fill the tank if both pipes were working?

2. Ken can fill a bin with pluots in 20 minutes. Kristen can fill the same bin with pluots in 25 minutes. If Ken and Kristen work together, how fast can they fill the bin?

3. In order to fill his pool Jared uses two hoses. Working alone, the first hose would take 10 hours to fill the pool and the second hose would take 15 hours to fill the pool. How long will it take with both hoses filling the pool?

4. Shawn usually mowes his lawn in 5 hours. Tracy can mow the same lawn in 4 hours. How long would it take them to mow the lawn working together?

5. The air conditioner in Mr. Blakely's office can cool the room 10 degrees in 90 min. Two doors down, Mr. Burch has an office of the same size but a nicer air conditioner. That air conditioner can cool down the room 10 degrees in 60 min. If we attach both air conditioners to Mr. Blakely's office, how long would it take to cool the room 10 degrees?

6. Paige can sort her toys in 6 fewer hours that Madison can. When they work together, it takes them only 4 hours to sort the toys. How long would it take for each of them to sort the toys alone?

7. Two painters, working together, can paint a house in 10 hours. Working alone, the first painter can paint the house in 15 hours. How long would it take for the second painter to paint the house working alone?

8. Working as a team, it takes Zoe and Haley 24 minutes to defeat the game *Math Attack*. Working by herself, it would take Zoe 36 minutes to beat the game. How long would it take Haley to beat the game by herself?

9. Zach can paint a house in 5 hours. Tyler can paint a house in 7 hours. If Zach and Tyler work together, how long should it take for them to paint 2 houses?

10. Jeff can help a customer with a new phone purchase 2 times faster than Lindsay can. When Jeff and Lindsay work together, they can help a customer with a new phone in 3 minutes. How long would it take for each of them to help the customer alone?

11. A company's new printer can print checks three times faster than its old printer. With both printers working, the checks can be printed in 6 minutes. How long would it take to print the checks with the new printer only?

12. When she works alone, Jennifer can feed all of the animals in the pet store 2 hours faster than James can. After working for 4 hours, James has to quit to help customers. At this point, Jennifer takes over feeding the animals. It takes her 3 hours to finish the feeding the animals. How long would it take each of them to feed the animals working alone?

13. Don can split a cord of wood in 6 fewer hours than MaryAnn. When the work together, they can split the cord in 4 hours. How long would it take each one to split a cord alone?

14. Mark can prune the trees at his house in 9 fewer hours than Matt can. When they work together, it only takes them 6 hours to prune the trees. How long would it take Matt to prune the trees alone?

15. When he works alone, Truda takes 4 hours longer than it takes Ivo to clean the bathrooms in the gym. After working for 3 hours, Truda quits for the day. If it takes Ivo 6 hours to finish cleaning the bathrooms in the gym, how long would it have taken Truda to clean the bathrooms by herself?

16. Dan and Brent, working together, can frame a house in 20 hours. Working alone, it would take Dan 30 hours longer than it would take Brent to frame the house. How long would it take each of them to frame the house alone?

17. Working together, it would take Scott and Patty 6 hours to clean their house. It would take Patty 3 times as long as it would take Scott to clean the house by herself. How long would it take each of them to clean the house alone?

18. Andrew can paint a painting 2 times faster than Ethan can. When they work together on a painting it takes them 9 hours to paint. How long would it take Ethan to paint the painting by himself?

19. Two identical printers are printing the payroll for College of the Sequoias. After they work together for 4 hours, one of the printers breaks down. The second printer takes 3 more hours to finish the job. How long would it take for one printer, working alone to print all of the payroll?

20. Two pipes are filling a pool with water. Working alone, the smaller pipe would take 3 hours longer to fill the pool than the larger pipe. After working for 2 hours alone, the small pipe breaks. It takes the larger pipe 2 hours to finish filling the pool. How long would it take the large pipe alone to fill the pool?

21. Checkers can catch and kill a single mouse in 40 seconds. Muffy can catch and kill a single mouse in 70 seconds. If Checkers and Muffy work together, how long would it take to catch and kill a nest of mice that contains 11 mice?

22. Stephanie can run the daily lab tests in 4.8 hours. Eric can do the job in 2.4 hours. How long would it take for them, working together, to run the daily lab tests twice?

23. To winterize a pool, two drains, working together can drain the pool in 3 hours. Working alone, the smaller drain would take 9 hours longer than the larger drain. How long would it take for the smaller drain to drain only half of the water out of the pool?

24. Rod can prepare a tax return in 15 hours. Christy can prepare a tax return in 20 hours. If Rod has been working of a tax return for 6 hours before Christy joins to help, how long should it take the two of them working together to finish the tax return?

25. Dave is training for a big race. Today, he ran 14 miles in 2 hours. After running the first nine miles at a certain speed, he increases his speed by 4 miles per hour. What is Dave's running speed for the first 9 miles?

26. Sue power walks 3 km/hour faster than Tim. In the time it takes Tim to walk 7.5 km, Sue walks 12 km. What is Sue's walking speed?

27. Eric's truck drives 30 mph faster than Tim's motorcycle. In the same time it takes Tim to drive 75 miles, Eric can drive 120 miles. Find Tim and Eric's driving speed.

28. Andy's tractor is just as fast as Dan's. It takes Andy 1 hour more to spray his trees than it takes Dan to spray his trees. If Andy drives 20 miles in his field to spray, and Dan drives 15 miles in his fields to spray, how long does it take for Dan to spray his trees?

29. A freight train leaving from Hanford is 14 mph slower than the passenger train which also leaves from Hanford. The passenger train travels 400 miles in the same amount of time that the freight train travels 330 miles. What is the speed of the passenger train?

30. It took Erin the same time to drive 21 miles as it took Brent to drive 15 miles. Erin's speed was 10 mph faster than Brents speed. How fast did Erin drive?

31. Jeff hiked to the top of Mt. Whitney from the trail head (a distance of 14 miles). Jon, after camping overnight on the trail, hiked only 12 miles to the top of Mt. Whitney. If Jeff and Jon were both hiking for the same amount of time, and Jon hikes 1 mph slower than Jeff, how fast did each man hike?

32. A load of stone fruit is being trucked to Los Angeles, CA from Kingsburg, CA. The truck is moving 40 mph faster than a train which is also going to Los Angeles. In the time it takes the train to travel 150 miles, the truck can travel a total of 350 miles. What is the trucks driving speed?

33. Colleen bicycles 6 km/hr faster than George. In the same time it take George to go 42 km, Colleen can go 60 km. How fast is Colleen?

34. Tom Farrell is training for another Olympic Medal. To do so, Tom swims 2 miles at a certain speed and then increases his pace by 2 mph and continues for another 12 miles. If Tom's workout lasted 4 hours, how fast was he swimming on each leg of the swim?

35. The rate of a motorcycle is 40 mph faster than the rate of a bicycle. The motorcycle travels 150 miles in the same amount of time it takes the bicycle to travel 30 miles. Find the rate of the motorcycle.

36. Terran bikes6 mph faster than his wife, Marria. In the same time that it takes Marria to bike 27 miles, Terran can bike 25 miles. What is Terrans biking speed?

37. A jet can fly at a rate of 250 mph in calm air. Traveling with the wind, the plane can fly 960 mi in the same amount of time in which it can fly 840 mi against the wind. Find the rate of the wind.

38. The speed of a stream is 2 mph. A rowing team travels 9 miles upstream in the same amount of time it takes to travel 13 miles downstream. What is the speed of the boat in still water?

39. Marty is an avid fisherman. While fishing in Montana, Marty takes a boat 10 miles upstream. Finding no fish upstream, Marty then travels 20 miles downstream. He found that it took him the same amount of time as the trip upstream. If the boat travels 15 mph, what is the speed of the current?

40. An airplane flies 495 miles with a tailwind then flies home. If the plane can fly 500 mph and the total trip took 2 hours, what is the speed of the wind?

41. An airplane flies 990 miles with a 50 mph tailwind and then flies back into the same wind. If the round trip was 4 hours, what is the speed of the plane with no wind?

42. The speed of a stream is 4 mph. A boat travels 11 miles upstream in the same amount of time it takes to travel 19 miles downstream. What is the speed of the boat in still water?

43. A boat can travels 30 miles up a river (against the current) in the same amount of time it takes to travel 50 miles down the river (with the current). If the current is 5 mph, what is the speed of the boat in still water?

44. An airplane flies 396 miles with a 40 mph tailwind and then flies home into the same wind. The total flying time was 2 hours. What is the speed of the plane?

45. A canoeist can paddle 8 mph in still water. Traveling with the current, the canoe traveled 30 mi in the same amount of time in which it traveled 18 mi against the current. Find the rate of the current.

46. Jennifer, an open water swimmer, is training for the Olympics. To do so, she swims in a stream that is 3 mph. Jennifer finds that she can swim 4 miles against the current in the same amount of time that she can swim 10 miles with the current. How fast can Jennifer swim with no current?

47. While taking a trip to Seattle, a plane flies against the wind 288 miles and then returns home with the same wind. In calm air, the plane can fly 300 mph. If the total flying time was 2 hours, what is the speed of the wind?

48. The speed of a stream is 2 mph. A boat travels 7 miles upstream and then 11 miles downstream. If the total trip took 2 hours, what is the speed of the boat in still water?

49. A salmon is swimming in a river that is flowing downstream at a speed of 2 miles per hour. The salmon can swim 12 miles upstream in the same amount of time it would take to swim 24 miles downstream. What is the speed of the salmon in still water?

50. An airplane flies 288 miles with a 60 mph tailwind and then flies back into the 60 mph wind. If the time for the round trip was two hours, what is the speed of the plane in calm air?