5.7 Literal Equations

Now that we have learned to solve a variety of different equations (linear equations in chapter 2, polynomial equations in chapter 4, and rational equations in the last section) we want to take a look at solving another type of equation which will draw upon all of the solving that we have learned about thus far.

We start with the definition.

> **<u>Definition</u>**: **<u>Literal Equation</u>**- an equation with two or more variables, also known as a "formula."

What we want to do with these equations is the same as what we always do with equations. We are interested solving them. However, since there are more than one variable, we are only interested in solving for a specified variable.

The good news is, we solve these with the same properties of solving that we use for any old equation. It's just that our goal here is to get the variable alone on one side, everything else, without the variable, on the other.

That is to say we want the equation to say **"variable = everything else without that variable"**. We use all of the same techniques and processes for solving that we have learned so far. It's just a matter of treating all of the extra variables as you would if they we simply numbers.

Let's take a look at some examples.

<u>Example 1</u>:

Solve for the indicated variable.

a. $P = 2l + 2w, \quad for\ w$ b. $A = \frac{1}{2}bh,\ for\ b$

Solution:

a. To solve the equation for *w*, we need to get the *w* all by itself on one side, and everything else on the other side. To do this, we get to use the same processes that we have always used.

$$P = 2l + 2w$$
$$-2l \quad -2l$$

Subtract 2l to get the term with the w alone

$$P - 2l = 2w$$

Divide by 2 to get w by itself

$$\frac{P - 2l}{2} = \frac{2w}{2}$$

Flip the sides to make the equation look like w = everything else

$$w = \frac{P - 2l}{2}$$

Since we have the *w* on one side and no *w*'s on the other side, this is our answer.

b. Again, to solve this equation, we want to get the *b* on one side, and everything else on the other. The difference here is that we are going to want to clear the fraction from the equation first. So we multiply by the LCD and then continue solving.

$$A = \frac{1}{2}bh$$

$$2 \cdot (A) = \left(\frac{1}{2}bh\right) \cdot 2$$

$$2A = bh$$

Divide by h to get b alone

$$\frac{2A}{h} = \frac{bh}{h}$$

$$b = \frac{2A}{h}$$

Since the *b* is alone, this is our answer.

Now that we have a basic idea of the process, let's look at some harder ones.

Example 2:

Solve for the indicated variable.

a. $A = \frac{1}{2}(b_1 + b_2)h,\ for\ h$

b. $A = \frac{1}{2}(b_1 + b_2)h,\ for\ b_2$

Solution:

a. Although the process for solving literal equation uses tools that we have picked up throughout the textbook, there are a few guidelines that we can follow when there are parenthesis involved in a literal equation.

   In this first one, the rule is, if the value you are looking to solve for is **outside** the parenthesis, as we can see *h* is here, then we **do not** distribute. It will be best to leave it outside.

   We start with, like in example 1b. clearing the fraction.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$2 \cdot (A) = \left(\frac{1}{2}(b_1 + b_2)h\right) \cdot 2$$

   Even though it is a little unusual to clear the fraction when you still have a grouping symbol, it works fine here because the 2 and the ½ cancel since on the right side of the equation everything is being multiplied (that is as long as we view the binomial $(b_1 + b_2)$ as one "unit". So this gives us

$$2A = (b_1 + b_2)h$$

   Clearly, now we need to only divide both sides by the $(b_1 + b_2)$ to get the *h* alone.

$$2A = (b_1 + b_2)h$$

$$\frac{2A}{(b_1 + b_2)} = \frac{(b_1 + b_2)h}{(b_1 + b_2)}$$

$$h = \frac{2A}{b_1 + b_2}$$

Since h is alone, we are done.

b. This time, notice the variable that we are solving for is **inside** the parenthesis. So, as we did before, we will start by clearing the fraction, but then, we will need to distribute to get the b_2 out of the parenthesis before trying to get it alone. It is always the case that it is better to get the variable you are solving for out of the grouping symbol before trying to solve it completely.

So, that being said, we clear the fraction and distribute.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$2 \cdot (A) = \left(\frac{1}{2}(b_1 + b_2)h\right) \cdot 2$$

$$2A = (b_1 + b_2)h$$

$$2A = b_1h + b_2h$$

Now get the b_2 alone.

$$\begin{array}{l} 2A = b_1h + b_2h \\ -b_1h \quad -b_1h \end{array}$$ Get the term with b_2 alone

$$2A - b_1h = b_2h$$

$$\frac{2A - b_1h}{h} = \frac{b_2h}{h}$$ Divide by h

$$b_2 = \frac{2A - b_1h}{h}$$

Finally, we are ready for some much more difficult literal equations.

<u>Example 3</u>:

Solve for the indicated variable.

a. $T = fm - gm,\ for\ m$ b. $\frac{1}{a} - \frac{1}{b} = \frac{1}{c},\ for\ b$ c. $R = \frac{1}{R_1} + \frac{1}{R_2},\ for\ R_1$

Solution:

a. As in the previous examples, we need to get *m* all by itself on one side of the equation, and everything else on the other. The issue here is that we have two *m*'s in the equation.

What we need to notice is that *m* can be factored out of the right side of the equation. This gives us

$$T = fm - gm$$
$$T = m(f - g)$$

Now we can divide by f-g to get the m alone. This gives

$$\frac{T}{f-g} = \frac{m(f-g)}{f-g}$$

$$m = \frac{T}{f-g}$$

b. With part a above in the back of our mind we will take a look at this next example. The first thing we should notice is that we have an equation containing fractions. Just like we always do, we will start by clearing the fractions by multiplying by the LCD which is abc.

$$\frac{1}{a}-\frac{1}{b}=\frac{1}{c}$$ Multiply by the LCD

$$abc\cdot\left(\frac{1}{a}-\frac{1}{b}\right)=\left(\frac{1}{c}\right)\cdot abc$$ Distribute the LCD

$$\frac{abc}{a}-\frac{abc}{b}=\frac{abc}{c}$$ Reduce

$$bc-ac=ab$$

At this point, we might be very tempted to simply divide both sides by *a* to get *b* all alone on the right side. However, that would leave an equation which has b on both sides. Remember, to solve for b mean "b = everything else **without b**".

So, instead we should get all the terms which have b in them onto the same side and everything without a "b" to the other side.

$$\begin{array}{r} bc-ac=ab \\ -bc \qquad -bc \end{array}$$ Subtract bc to get b's on one side

$$-ac=ab-bc$$ Factor out the b as we did in part a

$$-ac=b(a-c)$$

$$\frac{-ac}{a-c}=\frac{b(a-c)}{a-c}$$ Divide by a - c to get the b alone

$$b=-\frac{ac}{a-c}$$

c. This example is very similar to part b above. In fact the process will be merely identical. The main thing to remember is when a variable has a subscript (like R_1 and R_2), the different subscripts distinguish the variables apart from each other. So, in the case of this equation, R, R_1 and R_2 are all different variable and need to be treated as such.

With this now known, we can solve accordingly.

$$R=\frac{1}{R_1}+\frac{1}{R_2}$$ Multiply by the LCD (R_1R_2)

$$R_1R_2\cdot(R)=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\cdot R_1R_2$$ Distribute

$$R_1R_2R=\frac{R_1R_2}{R_1}+\frac{R_1R_2}{R_2}$$ Reduce

$$R_1R_2R=R_2+R_1$$

Again, we may be tempted to just move the R_2 over and call it done. However, we would have an R_1 on both sides. So instead we will have to collect the R_1 terms to one side and factor.

$$R_1R_2R = R_2 + R_1$$
$$-R_1 \qquad -R_1$$

Subtract R_1 from both sides

$$R_1R_2R - R_1 = R_2$$

Factor out R_1

$$R_1(R_2R - 1) = R_2$$

Divide to get the R_1 alone

$$\frac{R_1(R_2R-1)}{R_2R-1} = \frac{R_2}{R_2R-1}$$

$$R_1 = \frac{R_2}{R_2R-1}$$

5.7 Exercises

Solve for the indicated variable.

1. $A = l \cdot w,\ for\ w$
2. $d = r \cdot t,\ for\ t$
3. $V = 2\pi r^2 h,\ for\ h$
4. $V = \frac{1}{3}\pi r^2 h,\ for\ h$
5. $A = 2\pi r^2 + 2\pi rh,\ for\ h$
6. $A = 4lw + bh,\ for\ h$
7. $y - y_1 = m(x - x_1),\ for\ m$
8. $3x + 7y = 4,\ for\ y$
9. $L = a(1 + ct),\ for\ c$
10. $y = x(3 - yz),\ for\ z$
11. $N = a + b(2 + ct),\ for\ c$
12. $q + r(n - 4) = d,\ for\ n$
13. $s = a + (n - 1)d,\ for\ d$
14. $n = r + (a + b)c,\ for\ c$
15. $N = C - rC,\ for\ C$
16. $r = S - St,\ for\ S$
17. $L = \frac{1}{3}(act + a),\ for\ a$
18. $y = 12xy + 3xz - x,\ for\ x$
19. $x^2 - 2xy + y^2 = 0,\ for\ x$
20. $x^2 - y^2 = 0,\ for\ y$
21. $L = \frac{E}{R+K},\ for\ R$
22. $x = \frac{n}{m-k},\ for\ k$
23. $\frac{1}{r} = \frac{1}{a} - \frac{1}{b},\ for\ a$
24. $\frac{1}{r} = \frac{1}{a} - \frac{1}{b},\ for\ b$
25. $\frac{1}{x} = \frac{1}{y} - z,\ for\ x$
26. $\frac{1}{x} = \frac{1}{y} - z,\ for\ y$
27. $A = \frac{1}{B} + \frac{1}{C},\ for\ B$
28. $A = \frac{1}{B} + \frac{1}{C},\ for\ C$
29. $\frac{x}{A} + \frac{2}{B} = \frac{1}{2},\ for\ x$
30. $\frac{x}{A} + \frac{2}{B} = \frac{1}{2},\ for\ A$
31. $\frac{2C}{A} + \frac{1}{B} = 2,\ for\ C$
32. $\frac{2C}{A} + \frac{1}{B} = 2,\ for\ B$

33. $\frac{x}{yz} - \frac{1}{z} = \frac{2}{y},\ for\ x$

34. $\frac{a}{y} + \frac{1}{z} = \frac{x}{y},\ for\ a$

35. $\frac{2a+1}{b} + \frac{1}{c} = \frac{1}{3},\ for\ a$

36. $\frac{2a+1}{b} + \frac{1}{c} = \frac{1}{3},\ for\ b$