4.8 Applications of Polynomials

The last thing we want to do with polynomials is, of course, apply them to real situations. There are a variety of different applications of polynomials that we can look at. A number of them will not get treated until later in the text, when we have more tools for solving than we do now.

In the meantime, we still have plenty of applications to keep us busy. Let's look at some.

Example 1:

Jordan is 2 years older than his brother James. If the product of their ages is one less than five times the sum of their ages, how old are Jordan and James?

Solution:

The first thing we should do in a word problem is assign a variable. In this case, since we want to know Jordan and James' ages, we need to make one of their ages x.

To make it easy, let's just say x = Jordan's age. This means, since Jordan is 2 years older than James, x - 2 = James' age.

Now we just need to make an equation. Once we have an equation, we will solve the equation in order to solve the problem.

Since the product (which means multiply) of their ages is x(x - 2) and it says in the second sentence, "the product … is one less than five times the sum" we must have the equation

$$x(x-2) = 5(x+x-2) - 1$$

Product of the ages — 5 times — sum of the ages — 1 less than

Now that we have the equation, we just solve like we did in the last section.

$$x(x-2) = 5(x+x-2) - 1$$

$$x^2 - 2x = 5x + 5x - 10 - 1$$

$$x^2 - 2x = 10x - 11$$
$$-10x + 11 \qquad -10x + 11$$

$$x^2 - 12x + 11 = 0$$

$$(x-11)(x-1) = 0$$

$$x - 11 = 0 \qquad x - 1 = 0$$
$$+11 \quad +11 \qquad +1 \quad +1$$

$$x = 11 \qquad x = 1$$

So, since Jordan's age is x, Jordan is either 11 or 1 years old. However, since James is 2 years younger, Jordan can't be 1 because if he was, James would be -1, which is impossible.

So, Jordan must be 11 years old and James must be 9 years old.

Example 2:

The length of a rectangle swimming pool is 2 yards more than 2 times the width. The area of the pool is 144 square yards. Find the dimensions of the pool.

Solution:

Problems involving geometric figures are very common in algebra, as well as in life. So, as we always do in a problem involving geometry, we draw a picture to help us with generating our equation. We will also have to remember our geometry formulas.

So, since our length is 2 yards more than twice the width, our rectangle must look like

width = w

length = l = 2w + 2

Now, since we have been given information about the area of the rectangle, it would make sense to use the formula for the area of the rectangle. Recall, the formula for area of a rectangle is $A = l \cdot w$.

So we simple insert the length ($l = 2w + 2$), width (w) and area (144) into the formula and solve the resulting equation.

$$A = l \cdot w$$ Substituting

$$(2w + 2) \cdot w = 144$$ Multiply out sides

$$2w^2 + 2w = 144$$
$$-144 \quad -144$$ Get 0 on one side

$$2w^2 + 2w - 144 = 0$$ Factor out the GCF

$$2(w^2 + w - 72) = 0$$ Factor by trial factors

$$2(w - 8)(w + 9) = 0$$ Set each factor to 0

$$w - 8 = 0 \qquad w + 9 = 0$$
$$+8 \quad +8 \qquad -9 \quad -9$$ Solve for w

$$w = 8 \qquad w = -9$$

Since w represents the width of a rectangle, it can't be negative. So, w = 8 yds. Since we want both dimensions of the pool, we now need to find l. Remember, l = 2w + 2. So putting 8 in for w we get

$$\begin{aligned} l &= 2w + 2 \\ &= 2(8) + 2 \\ &= 16 + 2 \\ &= 18 \end{aligned}$$

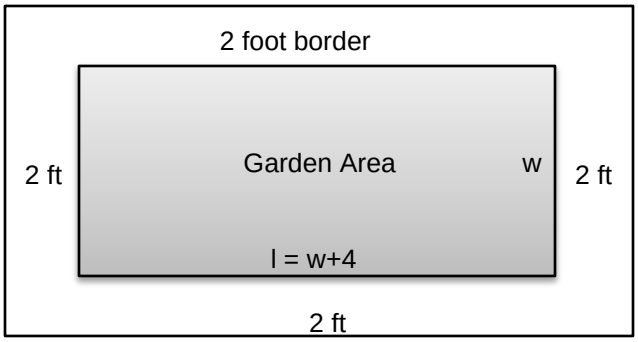
So the pool is 8 yds by 18 yds.

Example 3:

The length of a rectangular vegetable garden is 4 feet more than its width. After a 2 foot cement border is placed around the garden, the area of garden and border is 320 square feet. Find the original dimensions of the vegetable garden.

Solution:

Again here we are dealing with a geometric figure. It's a little bit harder this time, though because we have a rectangle, with a border around it. Here is the picture.



So, in this case, they gave us the area of the garden AND the border is 320 square feet. So, to use the area of the garden and border, we need the length and width of the garden and border.

By looking at our picture above, we can see that the length of the garden and border together is the length of the garden plus 2 feet on each side. This gives a length of

$$length\ of\ garden + border = w + 4 + 2 + 2$$
$$length\ of\ garden + border = w + 8$$

Similarly, the width would be the width of the garden plus 2 feet on each side. This gives

$$width\ of\ garden + border = w + 2 + 2$$
$$width\ of\ garden + border = w + 4$$

So we use these values to make our equation.

$$Area\ of\ garden + border = (length\ of\ garden + border) \cdot (width\ of\ garden + border)$$

$$(w + 8)(w + 4) = 320$$

Now we solve.

$$(w + 8)(w + 4) = 320$$

$$w^2 + 12w + 32 = 320$$
$$-320 \quad -320$$

$$w^2 + 12w - 288 = 0$$

$$(w - 12)(w + 24) = 0$$

$$w - 12 = 0 \qquad w + 24 = 0$$
$$+12 \ +12 \qquad -24 \ -24$$

$$w = 12 \qquad w = -24$$

As in example 2, widths can't be negative, so the width of smaller "garden only" area is 12 feet. We know this is just the garden because it was the only thing labeled "w" in our picture. So, since the length is 4 feet more than the width, the length must be 16 feet.

So our garden is 12 feet by 16 feet.

Example 4:

The formula for the number of games to be played in a football league where each team is to play each other twice is $N = x^2 - x$, where x is the number of teams in the league and N is the number of games to be played. If a league wants to limit the games to a total of 132 games, how many teams can be in the league?

Solution:

This time they have given us the formula for the given situation. So all we need to do is interpret the formula and plug in what they have given us.

According to the formula, N represents the number of games played. Since they have told us they want to schedule 132 games, N = 132. Once we plug that in, we merely need to solve the resulting equation.

$$N = x^2 - x$$

$$132 = x^2 - x$$

Plug in 132 for N

$$-132 \qquad -132$$

Get 0 on one side

$$x^2 - x - 132 = 0$$

Factor

$$(x - 12)(x + 11) = 0$$

Set each factor to zero and solve

$$x - 12 = 0 \qquad x + 11 = 0$$
$$+12 \quad +12 \qquad -11 \quad -11$$

$$x = 12 \qquad x = -11$$

Since x represents the number of football teams in a league, x cannot be negative. Therefore, the league needs to have 12 teams.

Example 5:

Standing on the top of a house 48 feet above the ground, Matt shoots a trick basketball shot upward according to the equation $h = -16t^2 + 32t + 48$, where h is the height of the ball in feet, and t is the time in seconds. How long will it take for Matt's airball to hit the ground?

Solution:

In a similar way to example 4, we have been given the formula for the situation. So we need to figure out what information we have been given and plug it into the formula and solve.

Unfortunately, we must be very careful to properly interpret what information has been given. Notice that even though they gave us a value of 48 feet above the ground, this information has nothing to do with the question that is being asked.

We want to know what happens when the ball **hits the ground**. Since h represents the height of the ball at any given time, we will have to know what the height is when something hits the ground.

Clearly, anytime an object hits the ground, its height above the ground is zero. So, we need to put h = 0 into the formula, and solve the equation. It's important to know that anytime we are dealing with an "object hitting the ground, the h = 0.

$h = -16t^2 + 32t + 48$ Set h = 0

$0 = -16t^2 + 32t + 48$ Factor out the GCF, -16

$0 = -16(t^2 - 2t - 3)$ Factor the trinomial

$0 = -16(t - 3)(t + 1)$ Set each factor containing variables to zero and solve

$t - 3 = 0 \qquad t + 1 = 0$
$+3 \quad +3 \qquad -1 \quad -1$

$t = 3 \qquad t = -1$

Since t represents time, it cannot be negative. Therefore, the time for the ball to hit the ground is 3 seconds.

Example 6:

While hunting, Scott R shoots an arrow from the top of a 23 foot platform at a trophy buck according to the equation $h = -16t^2 + 76t + 23$, where h is the height in feet and t is the time in seconds. Sadly, his arrow misses and hits a tree. As Scott arrives, he finds that the arrow hit the tree at a height of 3 feet off the ground. How long did it take for the arrow to hit the tree?

Solution:

This example is very similar to example 5. The difference here is that the formula itself is different, and this time we are not interested in finding the time required for the object to "hit the ground". Here, we are given a height of 3 feet off the ground.

So, plug 3 into the formula for h, and solve the resulting equation.

$h = -16t^2 + 76t + 23$ Set h = 3

$3 = -16t^2 + 76t + 23$ Get 0 on one side
$-3 \qquad\qquad -3$

$0 = -16t^2 + 76t + 20$

Factor out the GCF, -4

$0 = -4(4t^2 - 19t - 5)$

Factor the trinomial

$0 = -4(4t + 1)(t - 5)$

Set each factor containing variables to zero and solve

$4t + 1 = 0 \qquad t - 5 = 0$
$-1 \quad -1 \qquad +5 \quad +5$

$\frac{4t}{4} = -\frac{1}{4} \qquad t = 5$

$t = -\frac{1}{4}$

So, the time must be 5 seconds for the arrow to hit the tree.

4.8 Exercises

1. Two consecutive pages have a product that is 306. What are the page numbers?

2. Two consecutive positive even numbers have a product that is 360. What are the numbers?

3. One positive number is 3 more than another. Their product is 180. What are the numbers?

4. Two positive numbers have a product of 575. If the larger number is 2 more than the smaller, what are the numbers?

5. Andrew and Ethan are brothers. Andrew is older. The sum of the ages of Andrew and Ethan is 30. If the product of their ages is 216, how old are Andrew and Ethan?

6. The difference in Cameron and Avery's ages is 5. The product of their ages is 104. If Cameron is older, how old is Avery?

7. David's age is 7 years more than three times Bree's age. If the product of their ages is 110, how old is David?

8. Chris is 2 years older than Tom. The product of their ages is 574 more than the sum of their ages. How old is Tom?

9. Joshua is a year older than his sister McKay. The product of their ages is 33 more than 7 times Joshua's age. How old are Joshua and McKay?

10. Madison is 7 less than four times as old as Paige. The product of their ages is 47 more than the sum of their ages. How old are Madison and Paige?

11. The length of a rectangular flower garden is 5 feet more than its width. If the area of the garden is 104 square feet, what are the dimensions of the garden?

12. The area of the floor in a square room is 121 square meters. What is the length of the wall of the room?

13. The length of a Ping-Pong table is 2 feet less than twice the width. The area of the Ping-Pong table is 40 square feet. What are the dimensions of the table?

14. The area of a rectangular wall of a barn is 64 square feet. Its length is 8 feet longer than twice its width. Find the length and width of the wall.

15. The length of a rectangular plot of land is 10 yards more than its width. If the area of the land is 600 square yards, find the dimensions of the plot of land.

16. The area of a rectangular lawn is 1200 square feet. If the length of the lawn is 27 feet less than three times its width, what is the length and width of the lawn?

17. The width of a photo is 5 inches less than its length. A border of 2 inches is placed around the photo. The area covered by the photo and its border is 104 square inches. Find the dimensions of the photo itself.

18. The Jackson family Christmas photo is 5 inches less than its width. A 2 inch frame is placed around the photo. The area of the photo and frame is 66 square inches. What are the dimensions of the photo alone?

19. Jennifer's rectangular flower garden has a length that is 5 feet less than twice the width. A 4 foot brick border is added around the garden giving a total area of garden and border of 255 square feet. What is the size of the garden without the border?

20. The area of John's rectangular lawn and cement border is 1222 square feet. If the cement border is 2 feet wide, and the length of his lawn is 27 feet less than 3 times the width, what are the dimensions of John's lawn?

21. The width of Eric's small painting is 3 inches less than its length. A 1 inch border is put around the painting to protect the sides of his masterpiece. The area of the painting and the border is 108 square inches. What is the length and width of Eric's original painting?

22. Cathy's rectangular flower garden has a length that is 3 feet less than twice the width. After she has her husband, David, place a 5 foot brick border around the garden, the area of the garden and border is 210 square feet. Find the dimensions of the garden without the border.

23. Jon wants to put a 1 yard wide deck around his rectangular pool. Before putting the deck in, the length of the pool was four times the width. If the total area after the deck is put on is 154 square yards, what are the dimensions of Jon's pool without the deck?

24. The height of a triangular sail is 5 feet more than twice the base. If the sail is made of 174 square feet of fabric, what are the dimensions of the sail?

25. The height of a triangular flag is 5 inches more than twice its base. If the flag is made of 84 square inches of fabric, what are the dimensions of the flag?

26. The number of possible handshakes, H, within a group of n people is given by the formula $H = \frac{1}{2}(n^2 - n)$. How many people would there have to be in a group to have 435 handshakes?

27. The number of possible high fives, H, within a group of n people is given by the formula $H = \frac{1}{2}(n^2 - n)$. After the Angels won the world series, there were 66 high fives at Jon's house. How many people were at Jon's house?

28. A camera shop's daily profit is given by $P = 3x^2 - 3x + 10$, where x is the total number of cameras sold in the day. If they only had profit of $70 today, how many cameras did they sell?

29. A small appliance company's daily revenue is given by $R = 2x^2 - 14x$, where x is the total number of appliances sold in the day. If they only had revenue of $60 on Monday, how many appliances did they sell on Monday?

30. Two motor boats start at the same spot and travel apart from each other according to the formula $d = (x + 1)(x + 2)$, where d is in miles, and x is the time they are traveling apart in minutes. How long have they been traveling when the distance between them is 6 miles?

31. Jared's stock portfolio grows according to the formula $a = 2x^2 - 31x + 25$, where a is the amount in the portfolio, in thousands, and x is the amount of time the money has been in the portfolio, in months. When will Jared's portfolio be at a value of $10,000?

32. Mark's stock portfolio grows according to the formula $a = 2x^2 + 7x + 2$, where a is the amount in the portfolio, in thousands, and x is the amount of time the money has been in the portfolio, in months. When will Mark's portfolio be at a value of $6,000?

33. A ball rolls down a slope and travels a distance of $d = 6t + \frac{t^2}{2}$ feet in t seconds. How long does it take a ball to roll a distance of 14 feet?

34. A ball rolls down a slope and travels a distance of $d = 6t + \frac{t^2}{2}$ feet in t seconds. How long does it take a ball to roll a distance of 32 feet?

35. An open box with a square base has a surface area given by $S = x^2 + 4xh$, where x is the length of the base of the box and h is the height. If a box has a height of 2 feet and a surface area of 84 square feet, what is the length of the box?

36. The number of bridge pairs in a tournament where each pair plays every other pair exactly once is given by $G = p^2 - p$, where G is the number of games and p is the number of pairs. If a tournament has 90 games, how many pairs were in the tournament?

37. The height h in feet of a model rocket launched vertically upward from the top of a 16 foot tall bridge is given by $h = 32 + 16t - 16t^2$, where h is in feet and t is in seconds. How long will it take the rocket to strike the ground?

38. The height h in feet of a penny thrown vertically upward from the top of a 64-foot tall building is given by $h = 64 + 48t - 16t^2$, where t is in seconds. How long will it take the penny to strike the ground?

39. The height h in feet of a ball thrown vertically upward from the top of a 288-foot tall building is given by $h = 288 + 48t - 16t^2$, where t is in seconds. How long will it take the ball to strike the ground?

40. The height h in feet of an object thrown vertically upward from the ground is given by $h = 128t - 16t^2$, where t is in seconds. How long will it take the object to return to the ground?

41. The height h in feet of a projectile launched vertically upward from the top of a 96-foot tall platform is given by $h = 96 + 16t - 16t^2$, where t is in seconds. How long will it take the projectile to strike the ground?

42. The height h in feet of a projectile thrown vertically upward from the top of a 640-foot tall building is given by $h = 640 + 48t - 16t^2$, where t is in seconds. How long will it take the projectile to hit the ground?

43. As part of a civil war reenactment, Tristan shoots of a cannon that is sitting on the ground. The cannon ball's height, h, is given by $h = -16t^2 + 76t$, where t is in seconds. How long will it take for the cannon ball to hit the ground?

44. While hanging out on a 640 foot tall bridge, Shawn decides to throw his gum away by tossing it as high in the air as he can and seeing how long it takes to hit the ground below. His gum travels according to the height $h = -16t^2 + 48t + 640$, where t is in seconds. How long does Shawn's gum take to hit the ground?

45. While tossing the football with his kids, Jon throws a ball as hard as he can to try to catch the kids by surprise. The height of the ball, h, is given by $h = -16t^2 + 8t + 9$, where t is in seconds. One of the kids makes a diving catch and catches the ball at 1 foot off the ground. How long was the ball in the air before being caught?

46. For his 60th birthday, Rene goes sky diving. The height of Rene above the ground is given by $h = -16t^2 + 2600$, where t is in seconds. Rene deploys his chute at 1000 feet above the ground. How long was Rene sky diving before he deployed his chute?

47. Jordan and James are throwing water balloons off a bridge to try and hit swimmers in a river below. The height above the river, h, of the balloons is $h = -16t^2 + 40t + 210$, where t is in seconds. One of the balloons hits a person in a boat below. If the person in boat was 10 feet above the river, how long did it take the water balloon to hit the person?

48. Jeff is having a snack on top of a cliff. While eating, he accidently drops his water bottle. The bottle has height $h = -16t^2 + 64t + 600$, where t is in seconds. The bottle hits a small ledge below. The ledge is 88 feet above the ground. How long did it take for the bottle to hit the ledge?

49. Stephanie shoots a bb gun into the air from the back of a pickup truck. The height of the bb above the ground is given by $h = -16t^2 + 96t + 12$, where h is in feet and t is in seconds. After how much time is the bb at a height of 120 feet?

50. Tim shoots a model rocket from the top of an 8 foot platform. The height of the rocket is given by $h = -16t^2 + 64t + 8$, where t is in seconds. At what time is Tim's rocket at a height of 68 feet?