

4.1 Common Factors

Now that we are familiar with the basic operations on polynomials, we need to introduce one additional operation that can be done on polynomials. This operation is a vital part of algebra. We will spend this entire chapter dealing with it in detail.

To begin, we recall the distributive property from chapter 1, but we will write it in “reverse”:

$$a \cdot b + a \cdot c = a(b + c)$$

An example of using the “Reverse” distributive property would be

$$2x - 4 = 2 \cdot x - 2 \cdot 2 = 2(x - 2)$$

This idea (that is, the idea of “undoing” multiplication) is called **factoring**.

In the case seen above, 2 is called a **common factor**.

We are not just interested in factoring any old common factor. We actually are interested in factoring out the largest possible common factor that we can.

That is, we want to factor the **Greatest Common Factor** or **GCF**.

Factoring out a GCF, is the first step in the larger process of factoring which we discuss throughout this chapter. So, since it is the first in a longer process, we want to make sure we can do it correctly. Here are the steps we can follow

Factoring out the GCF
1. Find the GCF a. Find the GCF of the “numbers” That is, the GCF is the biggest number that goes evenly into all of the numbers b. Find the variable part of the GCF by choosing all the variables common to every term and raise each to the lowest individual power seen. c. The product of these is the GCF. 2. Rewrite each term so that it is the GCF times “other stuff”. 3. Use the distributive property to factor out the GCF.

Even though a few of the steps above sound vague, as we do some examples, it will become clear as to what we mean by “numbers” and “other stuff”.

Example1:

Factor.

- | | |
|----------------------------------|----------------------------|
| a. $24a^2 + 12a - 6a^3$ | b. $15L^2 - 21LW^2 + 36LH$ |
| c. $16x^3y^2 - 3x^2y^2 + 24xy^3$ | d. $3x^2 - 3xy + 3x$ |

Solution:

- a. The first thing we need to do is determine what the GCF is. So, as it states in the steps above, there are two pieces to the GCF, the “number” part, and the “variable” part.

The number part of the GCF is simply the GCF of the “numbers” in each term, meaning the biggest number that goes evenly into all of the coefficients and the constant terms (if any).

In this case, since the biggest number that goes into 24, 12 and 6 is 6, the “number” part of the GCF is 6.

Now, as for the variable part of the GCF, we need to choose all the variables that are common to all the terms, in this case, its just a . Then we raise it to its lowest individual power, in this case, the first power.

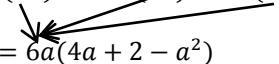
So, multiplying those two things together gives us a GCF of $6a$.

Now we need to rewrite each term so that it is the GCF times “other stuff”. What this means is each term needs to be written so that it is the GCF times whatever else is needed so that the new expression still has the same value.

For example, the first term is $24a^2$. So, we can rewrite this term as $(6a) \cdot 4a$ since it still gives me something that is equivalent to $24a^2$. With this in mind we write the expression as follows

$$\begin{aligned} 24a^2 + 12a - 6a^3 \\ = (6a) \cdot 4a + (6a) \cdot 2 - (6a) \cdot a^2 \end{aligned}$$

Now, by the “reverse” distributive property, we can factor the GCF out of the expression. Basically, this means we can remove the GCF part of each of the terms as write it out in front of the expression. This gives

$$\begin{aligned} (6a) \cdot 4a + (6a) \cdot 2 - (6a) \cdot a^2 \\ = 6a(4a + 2 - a^2) \end{aligned}$$


- b. Again, we start by finding the GCF. This time, the GCF of the numbers is 3 (the largest number that divides evenly into 15, 21 and 36) and the GCF of the variables is L . So the GCF is $3L$.

Now, rewriting each term as the GCF time “other stuff” gives

$$\begin{aligned} 15L^2 - 21LW^2 + 36LH \\ = (3L) \cdot 5L - (3L) \cdot 7W^2 + (3L) \cdot 12H \end{aligned}$$

Now factor out the GCF to get

$$= 3L(5L - 7W^2 + 12H)$$

- c. Starting again with finding the GCF, we notice that there is no number (other than 1) that goes evenly into all of the numbers. But there is an x and a y in common to all of the terms.

Therefore, choosing the lowest power of each of the x and y gives us the GCF of xy^2 .

So we rewrite each term and factor out the GCF by distributive property.

$$\begin{aligned} 16x^3y^2 - 3x^2y^2 + 24xy^3 \\ = (xy^2) \cdot 16x^2 - (xy^2) \cdot 3x + (xy^2) \cdot 24y \\ = xy^2(16x^2 - 3x + 24y) \end{aligned}$$

- d. Lastly, we can clearly see that the GCF must be $3x$. Notice, however, that our last term is exactly the same as the GCF. So when we go to rewrite our terms, what should we put? In a case like this, we must always remember that there are invisible 1's all over the place. So we can rewrite our expression and factor as follows

$$\begin{aligned} 3x^2 - 3xy + 3x \\ &= (3x) \cdot x - (3x) \cdot y + (3x) \cdot 1 \\ &= 3x(x - y + 1) \end{aligned}$$

Just a few things to take note of at this point. First of all, usually we don't show the step of rewriting the terms in factoring out a GCF. As we can see from the previous example, when factoring out a GCF, the answer always looks like

GCF(all the "other stuff" terms)

So we normally just cut to the chase and go directly from the problem to the answer. This is what we will do from now on.

The next thing to keep in mind is that when factoring out a GCF, the polynomial inside the parenthesis must have the same number of terms as the polynomial you started with. Keeping this in mind will help you to remember to keep a 1 in the place where a term factoring completely out as a GCF.

Lastly, as it turns out, we are not only limited to a monomial GCF. The next example will show you that we can also have a binomial, or any size polynomial for that matter, GCF. The process of factoring it out is the same.

Example 2:

Factor.

a. $3x(2x - 1) - 4(2x - 1)$

b. $15x(2a + b) + 10y(2a + b)$

Solution:

- a. The first thing we need to address is that this polynomial only (technically) has two, rather large, terms. They are, $3x(2x - 1)$ and $4(2x - 1)$. So, if we keep this in mind, clearly these terms have what is called a **common binomial factor**.

Since the binomial is common to the terms, it must be part of (and in this case the entire) GCF. So we factor it out, just as we would do if it was simply an x , or y , or 2 , etc.

That is to say, the GCF comes out front, and what is left over from each term is left inside the parenthesis. This gives us

$$\begin{array}{c} 3x(2x - 1) - 4(2x - 1) \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ = (2x - 1)(3x - 4) \end{array}$$

- b. Just like part a above, we clearly see a binomial GCF. However, the binomial is not the entire GCF here. If we look hard enough, we also have a common 5 to our large terms.

This means our GCF is actually $5(2a + b)$.

So, factoring the GCF out gives us

$$\begin{aligned} &15x(2a + b) + 10y(2a + b) \\ &= 5(2a + b)(3x + 2y) \end{aligned}$$

Be very careful when factoring out a binomial GCF. The common error is to factor it out of each term and try to assign a power to it. Always remember, you factor a binomial out the same way you factor an x out. You simply remove it from each term and only one of them appears in front of the expression.

4.1 Exercises

Factor out the GCF.

1. $4x - 16$
2. $3x + 9$
3. $35x^2 - 15x$
4. $49y^2 + 14y$
5. $6a^2 + 3a$
6. $10x^2 - 5x$
7. $15x^2 - 5xy + 10x$
8. $45a^2 + 15a - 9$
9. $45x^6 - 35x^4 - 75x^3$
10. $15x^7 - 33x^6 - 75x^2$
11. $m^7n^3 - m^5n^6 + m^7n^5$
12. $a^5b^3 - a^6b^6 + a^7b^2$
13. $2x^4 - 2x^3 - 2x$
14. $3x^2 - 3x^4 - 3$
15. $6a^2b^2 - 9a^2b + 12ab^2$
16. $12a^2b^2 + 18a^2b - 24b^2$
17. $9x^6y^3 - 36x^4y^5 + 45x^6y^5$
18. $4x^5y^6 - 36x^8y^3 + 16x^6y^5$
19. $28x^7 - 35x^4 - 98x^3$
20. $36b^5 - 24b^4 - 48b^3$
21. $63x^2y + 42xy + 21y$
22. $6x^2y^2 + 2xy + y$
23. $16p^2q^3 - 24p^3q^3 + 12pq^2$
24. $n^5m^4 - 7n^5m^6 + 5n^8m^5$
25. $20x^7y - 15x^4y - 50x^3y$
26. $4q^6p^3 - 3p^2q^7 + 12p^3q^6$
27. $2L^7W^3 - 6L^4W^5 + 3L^6W^6$
28. $x^2y^2z^2 - x^3y^3z - x^5y^7z$
29. $9x^7y^4z - 27x^5y^6z + 45x^8y^7z$
30. $26x^6y^3 - 39x^7y^3 + 52x^2y^7$
31. $60x^4y + 30x^3y + 20x^2y - 100xy$
32. $25x^3y^2 + 50x^4y + 100x^2y^2 - 125xy$
33. $ab - ac + ad - a^2 - a$
34. $xyz - xy - xz - x^2 - 2x$
35. $6x^7y^3 - 12x^5y^6 + 30x^8y^6 - 6x^5y^3$
36. $8x^5y^4 + 12x^3y^6 - 20x^2y^4 - 32x^8y^2$
37. $x(4x - 5) - 8(4x - 5)$
38. $x(2x - 3) - 6(2x - 3)$
39. $6a^3(3a + 2) + 7(3a + 2)$
40. $x^2(2x + 1) + 2(2x + 1)$
41. $2x(2x - 3) - 9(2x - 3)$
42. $2y(2y - 1) - 7(2y - 1)$
43. $4x^2(x - 2) - 5(x - 2)$
44. $6b^2(b + 2) - 7(b + 2)$
45. $8x(2x + 9) - (2x + 9)$
46. $x(x + 1) + (x + 1)$
47. $4y(4y - 1) - (4y - 1)$
48. $4x(y - 1) - (y - 1)$

49. $4x(3x + 4) + 8(3x + 4)$

51. $5xy(x - 2) - 3x(x - 2)$

53. $2x^2y(2x + 3) - xy(2x + 3)$

55. $3x(x - 1)^2 + 2(x - 1)^2$

57. $7x(x + 7)^2 + 4(x + 7)$

59. $x(x + 2)^4 + 2(x + 2)^3$

50. $4a(a - 1) + 6(a - 1)$

52. $ab(c - 2) - 2a(c - 2)$

54. $7p^2q(4n - 3) - pq(4n - 3)$

56. $x(3x - 1)^2 - 5(3x - 1)^2$

58. $x(x - 4)^2 - 4(x - 4)$

60. $5x(x - 2)^3 + 3(x - 2)^2$