3.6 Synthetic Division

Having now learned how to long divide polynomials, we can see that the process, although accurate, is very tedious and time consuming.

Fortunately, there is a much more efficient way to divide certain types of polynomials. Whenever we are dividing by a first degree binomial (recall that means two terms of which the power on the variable is one) we can use a process called synthetic division to do so. It is a much simpler and concise process for polynomial division.

Be warned… Synthetic Division only works when dividing by a first degree binomial. Any other case of division requires long division like we learned in the previous section.

The simplest way to learn synthetic division is to learn by way of examples.

<u>Example 1</u>:

Divide using synthetic division.

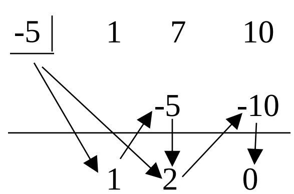
a. $\frac{x^2+7x+10}{x+5}$ b. $(x^2 + 6x + 10) \div (x + 3)$ c. $(x^4 - 14) \div (x - 2)$

Solution:

a. First, we need to set up our synthetic division. The way it works is you start by taking what would be the solution to the polynomial that you are dividing by (which basically means change the sign of the constant term) and place it in a box in the upper left and taking the coefficients of the divisor and putting them along side the box in order. It looks like

-5	1	7	10

Then, the way it works is, you pull straight down the first number, in this case the 1. Then, multiply the number in the box times the number on the bottom. Place that number in the open space in the next column. Add vertically, and then multiply again and repeat until you run out of terms to add vertically. Here is what it looks like.



Multiply by -5 on each diagonal
Add the numbers vertically

What we end up with is the string of numbers 1, 2, 0. The last number is always the remainder term, like we learned in the last section. The first numbers are the coefficients of the answer starting with the degree one less than what you were dividing to begin with. So, 1, 2, 0 turns into $1x + 2 + \frac{0}{x+5}$.

So our answer is simply x + 2.

b. Again, we need to start by setting ourselves up correctly first. So, in the box would go -3 since it's the solution to the equation we would get from the binomial we are dividing by. We then list the coefficients in order.

$$\begin{array}{r|rrr} -3 & 1 & 6 & 10 \\ & & & \\ \hline & & & \end{array}$$

Then we begin the process by bringing down the 1 and multiplying each number that shows up under the line by -3 and placing that answer under the next column and adding. We get

$$\begin{array}{r|rrr} -3 & 1 & 6 & 10 \\ & & -3 & -9 \\ \hline & 1 & 3 & 1 \end{array}$$

So, the 1 in the back is the remainder term, and the 1 and 3 in the front are the coefficients of the answer starting with x since that is one degree lower than the polynomial we were dividing into.

Therefore we get $x+3+\frac{1}{x+3}$

c. This time notice we are missing a few terms. This means we will have to insert a 0 for every term we are missing, quite similar to what we did in the previous section. This gives us,

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -14 \\ & & & & & \\ \hline & & & & & \end{array}$$

Continuing as we did for the previous examples we get

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -14 \\ & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & 2 \end{array}$$

So starting with x^3 we place the numbers on the bottom row as the coefficients of our answers and make the 2 the numerator of the remainder term.

So our answer is $x^3+2x^2+4x+8+\frac{2}{x-2}$

Notice that in all of the previous examples, the leading coefficient on the binomial is 1. So, the question becomes, what do we do if the leading coefficient is NOT 1. The next example will help us through this very thing.

<u>Example 2</u>:

Divide using synthetic division.

a. $\frac{2x^2-7x-14}{2x-1}$

b. $(9x^3+6x^2-5)\div(3x+2)$

Solution:

a. The key to synthetic division is that it only work when dividing by a binomial with leading coefficient of 1. So, as in this example, when the leading coefficient is not 1, we start by dividing the leading coefficient (2) out of every single term in the entire division problem. This gives us,

$$\frac{2x^2 - 7x - 14}{2x - 1}$$

$$= \frac{x^2 - \frac{7}{2}x - 7}{x - \frac{1}{2}}$$

Although this seems quite improper, and quirky, we are now set up to synthetically divide as we did in example 1. We get

$$\begin{array}{r|rrr} \frac{1}{2} & 1 & -7/2 & 7 \\ & & 1/2 & -3/2 \\ \hline & 1 & -3 & 11/2 \end{array}$$

So, putting this in the polynomial give us

$$x - 3 + \frac{\frac{11}{2}}{x - \frac{1}{2}}$$

Clearly this needs to be simplified so that we don't have fractions inside of the remainder fraction. The way we take care of this is to simply multiply each term in the numerator and denominator by the LCD of 2. This will give us a much more simplified form. So we get

$$x - 3 + \frac{\frac{11}{2} \cdot 2}{x \cdot 2 - \frac{1}{2} \cdot 2}$$

$$= x - 3 + \frac{11}{2x - 1}$$

b. We start like we did in part a. Divide each term by 3 so that we can have a leading coefficient of 1. This gives us

$$(9x^3 + 6x^2 - 5) \div (3x + 2)$$

$$= \left(3x^3 + 2x^2 - \frac{5}{3}\right) \div \left(x + \frac{2}{3}\right)$$

Then divide using synthetic division. We need to make sure we put our 0 placeholder in for the x term since there is not one there.

$$\begin{array}{r|rrrr} -\frac{2}{3} & 3 & 2 & 0 & -5/3 \\ & & -2 & 0 & 0 \\ \hline & 3 & 0 & 0 & -5/3 \end{array}$$

So this gives us

$$3x^2 + \frac{-\frac{5}{3}}{x + \frac{2}{3}}$$

Simplifying by multiplying the remainder fraction by 3 we get

$$3x^2 - \frac{5}{3x+2}$$

3.6 Exercises

Divide by using synthetic division.

1. $\frac{x^2+9x+20}{x+4}$
2. $\frac{x^2-18x+81}{x-9}$
3. $(x^2 - 13x + 36) \div (x - 9)$
4. $\frac{6x^2-4x+5}{x-1}$
5. $\frac{x^2+5x-30}{x+8}$
6. $\frac{3x^2-10x+1}{x-4}$
7. $\frac{12x^2-30x+36}{3x+3}$
8. $(x^2 + 4x + 4) \div (x + 2)$
9. $\frac{x^3-x^2+x-6}{x+1}$
10. $\frac{6x^2-20x+125}{2x+8}$
11. $(x^3 - x^2 - x - 3) \div (x + 1)$
12. $\frac{36x^3-64x^2-88x-16}{9x+2}$
13. $\frac{2x^3+3x^2+5x-8}{x+2}$
14. $\frac{x^3+4x^2+x-24}{x+7}$
15. $\frac{4x^3-8x^2+7x-2}{2x-1}$
16. $\frac{x^4-x^3-x^2-x+1}{x-2}$
17. $\frac{3x^3-7x^2-22x+8}{3x-1}$
18. $\frac{36x^3-64x^2-88x-16}{9x+2}$
19. $\frac{21x^3-29x^2-24x-4}{7x+2}$
20. $\frac{3x^3-4x^2+x-17}{x-4}$
21. $(x^2 - 16) \div (x + 4)$
22. $\frac{x^3+64}{x+4}$
23. $(x^3 - x + 6) \div (x - 3)$
24. $\frac{x^3+7x-44}{x+9}$
25. $\frac{4x^3-29x-38}{2x+3}$
26. $\frac{x^3-x+30}{x-6}$
27. $\frac{x^4+2x^2-60x+26}{x-4}$
28. $\frac{27x^3-39x-26}{3x+2}$
29. $\frac{x^4+15x^2-19x+93}{x-2}$
30. $(x^5 + 2) \div (x + 3)$