

3.3 Introduction to Polynomials

At this point, we need to turn our attention to a subject that will carry us through a large chunk of the remainder of the first year of algebra, polynomials.

What is a polynomial, let's take a look.

Definition: Polynomial- an expression with real number coefficients, natural number powers, and no division.

So, this means a multi-termed variable expression with whole number powers and coefficients which are numbers we recognize.

For example, $x^3 + zxy - 7y^2 + 48 - m$ is a polynomial.

Much of the time, polynomials are referred to by the number of terms which they have. For example, $2x^2$ has only 1 term, so we call it a **monomial**. Likewise, $3x + y$ has two terms and is called a **binomial**, and $x^2 - x - 1$ is a **trinomial** since it has 3 terms.

Polynomials which are larger than 3 terms are simply referred to by the number of terms they have, that is, "a 4-term polynomial, a 5-term polynomial, etc".

Now we must be careful with our definition above, the fact that it has "no division" does not mean we can't divide two polynomials. It just means that if we happen to have division, we no longer have a polynomial, we would simply have a variable expression.

In this section we will deal with much of the foundations of these polynomials as well as begin to work on our operations on polynomials.

Let's start with the following definitions.

Definitions:

Leading term- the term in a polynomial with the highest power

Leading coefficient- the coefficient of the leading term

Degree of a polynomial- the power of the highest power term

Descending order- writing a polynomial so that the degree of the terms gets smaller

To get a handle on these definitions let's look at the following example.

Example 1:

Find the leading term, leading coefficient, and degree.

a. $14x^2 + y$

b. $-3x^2 - 5x + 8$

c. $4x + x^5 - 2x^3$

d. $x^2 - 3x^3 + 1 - x^4$

Solution:

- a. The first thing we should do is make sure that the polynomial is in descending order. This always makes identifying the information easier.

To get it in descending order, we just need to make sure the degree (that is, the power) of each term gets smaller as we move from left to right. In this case, the polynomial is already in descending order.

Once this is the case, the leading term is the term in front (hence the name “leading” term), the leading coefficient is the coefficient of that leading term, and the degree is the power of that leading term.

So we have the leading term is $14x^2$, the leading coefficient is 14 and the degree is 2

- b. Again, notice that the polynomial is already in descending order. Therefore, the leading term is $-3x^2$, the leading coefficient is -3 and the degree is 2.
- c. This time, the polynomial is not in descending order. So we must start by rearranging the terms so that it is in descending order. We do so as follows

$$\begin{aligned} &4x + x^5 - 2x^3 \\ &= x^5 - 2x^3 + 4x \end{aligned}$$

Now the leading term is truly “leading” the polynomial. So we see the leading term is x^5 , the leading coefficient is 1, and the degree is 5.

- d. Lastly, we again need to put the polynomial into descending order.

$$\begin{aligned} &x^2 - 3x^3 + 1 - x^4 \\ &= -x^4 - 3x^3 + x^2 + 1 \end{aligned}$$

So, the leading term is $-x^4$, the leading coefficient is -1, and the degree is 4

A couple of notes about the previous example:

If there happens to be several variables within the terms of the polynomial, then the degree would simply be the sum of the powers of those variables.

Also, if there are two (or more) terms that have the same degree, and it happens to be the highest degree, then the polynomial would have two leading terms.

Now that we have a good idea of some of the basics of polynomials, we need to start working on the basic operations on polynomials.

We start with the simplest operations in this section, that is adding and subtracting, and treat the other operations in the sections and chapters to follow.

Adding and Subtraction Polynomials

Simply combine like terms as we have done in the past.

So, what this means is, we already know how to do adding and subtracting polynomials. It's the same as combining like terms we learned in chapter 1, we are simply calling it “adding and subtracting polynomials” now.

Example 2:

Perform the operations.

a. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

b. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$

c. $(-2xy^2 - 4yz - 12) + (5xy^2 - 5yz)$

d. $(7x^4 + 8x^3 - 4x^2 - 3x + 1) - (6x^4 + 7x^3 - 3x^2 - 2x + 1)$

Solution:

- a. To add two polynomials we simply need to combine like terms like we learned how to do in chapter 1. Notice, the parenthesis in the problem are not doing anything. Therefore, we can simply get rid of them.

$$\begin{aligned}(3x^2 - 2x + 7) + (-3x^2 + 2x - 12) \\ &= 3x^2 - 2x + 7 + -3x^2 + 2x - 12 \\ &= -5\end{aligned}$$

- b. To subtract two polynomials, we need to, first of all, remember to distribute the negative through the second polynomial. Then combine like terms as before.

$$\begin{aligned}(3a^2 - 9a) - (-5a^2 + 7a - 6) \\ &= 3a^2 - 9a + 5a^2 - 7a + 6 \\ &= 7a^2 - 16a + 6\end{aligned}$$

- c. Again, we add by combining like terms.

$$\begin{aligned}(-2xy^2 - 4yz - 12) + (5xy^2 - 5yz) \\ &= -2xy^2 - 4yz - 12 + 5xy^2 - 5yz \\ &= 3xy^2 - 9yz - 12\end{aligned}$$

- d. Lastly, always remember to distribute the negative in order to subtract two polynomials.

$$\begin{aligned}(7x^4 + 8x^3 - 4x^2 - 3x + 1) - (6x^4 + 7x^3 - 3x^2 - 2x + 1) \\ &= 7x^4 + 8x^3 - 4x^2 - 3x + 1 - 6x^4 - 7x^3 + 3x^2 + 2x - 1 \\ &= x^4 + x^3 - x^2 - x\end{aligned}$$

3.3 Exercises

Find the leading term, leading coefficient, and degree.

1. $2x^2 - 3x - 5$

2. $6x^2 + 5x - 5$

3. $16x^6 - 49$

4. $x^2 - 3x$

5. $x^2 - 12x + 36$

6. $-x^2 - x - 15$

7. $-6x^4 + 9x^3 - 2x^2 - 7x - 2$

8. $19x^3 - 12x^2 - x - 2$

9. $9 - x^{12} + 12x$

10. $x^2 + 3x^7 - 7x - 2$

11. $3a - 5a^7 - 8a^3$

12. $2y^4 + 9y^6 - y^3 - 5y - 2$

13. $29 - 5b^8 + 6b - 4b^6$

14. $3n^4 + 2n^5 - n - 1$

15. $3x^2 - 17x + 3x^5 + 13$

16. $c^4 + c^8 - 2c^2 - c^5 - 2$

17. $5y^2 + 3y - 4y^8 - 9y^3 - 3y - 5$

18. $16x^4 + 5x^3 - x^7 - 6x$

19. $4x + 9x^7 - x^9 - 7x^5$

20. $6x^4 + 6x^3 - x^2 - 7x - 2x^{12} - 5$

Perform the operations.

21. $(6x^2 - 7x + 11) + (4x^2 + 9x - 62)$

22. $(5x^2 - 6x + 16) + (x^2 - x - 2)$

23. $(3x^2 + 3x - 7) - (2x^2 + 3x + 7)$

24. $(-2x^2 - 4x + 1) - (9x^2 + 8x - 6)$

25. $(4x^2 - 2x + 1) - (4x^2 - 3x + 1)$

26. $(2x^2 - 5x + 3) + (x^2 + 3x - 3)$

27. $(2x^3 + 7x^2 + x) + (x^2 + 5x - 11)$

28. $(6x^3 + 7x^2 + 2x) - (4x^2 + 9x - 1)$

29. $(7x^3 - 3x + 6) - (7x^3 - 2x + 6)$

30. $(x^5 - 7x + 11) + (x^5 + x - 5)$

31. $(2x^9 - 7x^4 + 7x^2) - (-6x^6 + 4x^5 - 13)$

32. $(-x^4 - x^2 + x) - (4x^2 + 9x - 12)$

33. $(x^3 - 8x - 32) + (5x^3 + x^2 + 2x + 26)$

34. $(2x^3 - 18x - 2) + (x^3 + 3x^2 + 2x + 6)$

35. $(10x^2y - 3xy^2 + 5xy) + (-7x^2y - 5xy^2 + 2xy)$

36. $(x^2y + xy^2 + 15xy) + (3x^2y - 6xy^2 - 2xy)$

37. $(5ab - 9a^2b + 13ab^2) - (9ab^2 - 3ab - 12a^2b)$

38. $(6ab - 4a^2b + 3ab^2) - (-3ab^2 + 8ab - a^2b)$

39. $(14x^2y - 4xy^2 + 6xy) + (-10x^2y - 6xy^2 + 4xy)$

40. $(6x^2y + 3xy^2 - xy) + (2x^2y - 3xy^2 + xy)$

41. $(2x^3yz^2 - 2xy^4z^3 - 14x^2y^2z^5) - (3x^2yz^3 + 14xy^4z^3 - 3x^2y^2z^5)$

42. $(3x^2yz - xy^4z^3 - 4x^2y^2z^5) - (13x^2yz + 4xy^4z^3 - 23x^2y^2z^5)$

43. $(5xyz^2 - 2xy^4z^3 - 10x^2y^2z^5) - (4x^2yz + 15xy^4z^3 - 4x^2y^2z^5)$

44. $(12x^3yz^2 + 12xy^4z^3 + 12x^2y^2z^5) - (13x^2yz^3 + 13xy^4z^3 + 13x^2y^2z^5)$

45. $(6ab - 8a^2b + 15ab^2) - (11ab^2 - 2ab - 15a^2b) - (8ab + 7a^2b + 4ab^2)$