

3.2 Scientific Notation

In the last section we learned all about the properties of exponents. In this section we want to turn our attention to an application of these properties of exponents.

In real life we need to be able to deal with very large or very small numbers.

For example, a cell could be 0.000000000000000000789 meters in diameter, or a country could have \$54,000,000,000,000 of debt.

We want to be able to work with these numbers and perform operations on them without having to deal with all of the zeros involved. The reason for that is the simple fact that when trying to work with all of the zeros, there is a big chance we would lose one of them in the process.

So, for this reason, we have a method by which we can make these very large, or very small numbers easier, and more concise, to write. Thereby making the operations on them much simpler.

This is called writing a number in scientific notation.

Definition: Scientific Notation- a number of the form

$$a \times 10^n, \text{ where } 1 \leq a \leq 10 \text{ and } n \text{ is an integer}$$

The first thing we need to do is be able to put numbers into, and take numbers out of scientific notation.

We will start with taking a number and writing it in scientific notation.

To write a number in scientific notation

1. Move the decimal so that it is between the first two digits, this number is "a"
2. Count the number of times you moved the decimal, this number is "n" with
 - a. "n" is positive if you started with a large number (bigger than 10)
 - b. "n" is negative if you started with a small number (smaller than 1)

Example 1:

Write in scientific notation.

- a. 0.0000081 b. 65,000,000 c. 0.000000000000000000789 d. 54,000,000,000,000

Solution:

- a. To put the number in scientific notation, we follow the steps outlined above. We start by moving the decimal so that it is between the 8 and 1. This gives us an "a" of 8.1. Now, since we moved the decimal 6 times and we started with a small number, our power of ten must be -6. So this gives us

$$8.1 \times 10^{-6}$$

- b. This time, we start by putting the decimal between the 6 and 5, giving us 6.5. We moved the decimal 7 times and the number was a large number. So our power of ten is 7. This gives

$$6.5 \times 10^7$$

- c. As above we can see our “a” value is 7.89 and counting we get the power of ten is -19. This gives us

$$7.89 \times 10^{-19}$$

- d. Finally, our “a” value is 5.4 and counting the times we moved the decimal gives 13 times. So we have

$$5.4 \times 10^{13}$$

Now that we can write a number in scientific notation, writing it back in decimal form is fairly easy. Here is the process.

To write a number in decimal notation

When the power of 10, n, is

1. Positive- move the decimal to make the number bigger (to the right), “n” places
2. Negative- move the decimal to make the number smaller (to the left), “n” places

Example 2:

Write in decimal notation.

- a. 2.3×10^5 b. 7.611×10^{-7} c. 6.52×10^{-5} d. 3×10^7

Solution:

- a. To put the number back in decimal notation, we simply have to move the decimal, 5 times, to make the number larger... to the right. We get

$$230,000$$

- b. Here, since the exponent of ten is negative, we move the decimal to make the number smaller, to the left, 7 times. We get

$$0.0000007611$$

- c. As in part b, we move the decimal to the left, to make the number small, 5 times. This gives

$$0.0000652$$

- d. Finally, like in part a, we need to move the decimal 7 times to the right to make the number large. We get

$$30,000,000$$

Now that we can convert back and forth between decimal and scientific notation, we need to be able to perform computations by using scientific notation, and computations on numbers in scientific notation.

Operations in Scientific Notation

1. Perform the calculations on the “number” parts in the front of the scientific notation numbers.
2. Use rules of exponents on the 10^n parts of the numbers in scientific notation.
3. Make sure your answer is in scientific notation, if not, adjust the “number” part and the power of ten accordingly.

Let's try this in the next example.

Example 3:

Perform the operations. Leave your answer in scientific notation.

a. $(1.87 \times 10^7)(5.26 \times 10^7)$

b. $(0.00000000034)^2$

c. $\frac{(39,600)(0.00264)}{(0.0000132)(66,000,000)}$

d. $\frac{320,000,000}{(0.000016)(0.00004)}$

Solution:

- a. The first thing we should do is reorganize the problem so that the “number” parts are together, and the “ten to a power” parts are together.

$$\begin{aligned} & (1.87 \times 10^7)(5.26 \times 10^7) \\ &= (1.87 \cdot 5.26) \times (10^7 \cdot 10^7) \end{aligned}$$

Now, the front part we can calculate using a calculator and get 9.8362. This value is our new “a” value.

In the back part, we simply add the exponents as we did in the last chapter. This gives us an answer of

$$9.8362 \times 10^{14}$$

- b. The first thing we should do here is put the number into scientific notation. We get

$$\begin{aligned} & (0.00000000034)^2 \\ &= (3.4 \times 10^{-10})^2 \end{aligned}$$

Now, using the properties of exponents, we can pull the squared through to each value inside. This means we need to square the first value, which gives 11.56, and multiply the exponents in the back, giving an exponent of -20. So we have

$$11.56 \times 10^{-20}$$

However, notice this is not in scientific notation, since the number in front is not between 1 and 10. Therefore, we need to move the decimal once to the left, and then adjust the exponent of 10 up by one since we were working with a number larger than 10.

We have

$$1.156 \times 10^{-19}$$

- c. Again, let's start by putting all of the numbers into scientific notation. We get

$$\begin{aligned} & \frac{(39,600)(0.00264)}{(0.0000132)(66,000,000)} \\ &= \frac{(3.96 \times 10^4)(2.64 \times 10^{-3})}{(1.32 \times 10^{-5})(6.6 \times 10^7)} \end{aligned}$$

Now, reorganize so that all of the decimal “a” parts are together and all of the “ten to a power” parts are together.

$$\frac{(3.96 \times 10^4)(2.64 \times 10^{-3})}{(1.32 \times 10^{-5})(6.6 \times 10^7)}$$

$$= \frac{3.96 \cdot 2.64}{1.32 \cdot 6.6} \times \frac{10^4 \cdot 10^{-3}}{10^{-5} \cdot 10^7}$$

Putting the front in a calculator gives 1.2. Also, we use properties of exponents on the back to get

$$1.2 \times \frac{10^4 \cdot 10^{-3}}{10^{-5} \cdot 10^7}$$

$$= 1.2 \times \frac{10^1}{10^2}$$

$$= 1.2 \times 10^{-1}$$

d. Lastly, we proceed as we did in part c.

$$\frac{320,000,000}{(0.000016) \cdot (0.00004)}$$

$$= \frac{3.2 \times 10^8}{(1.6 \times 10^{-5})(4 \times 10^{-5})}$$

$$= \frac{3.2}{1.6 \cdot 4} \times \frac{10^8}{10^{-5} \cdot 10^{-5}}$$

$$= 0.5 \times \frac{10^8}{10^{-10}}$$

$$= 0.5 \times 10^{18}$$

$$= 5 \times 10^{17}$$

Change to sci. notation

Reorganize

Put front in calculator
Properties of exponent in
back

Put answer in sci. notation

3.2 Exercises

Write in scientific notation.

- | | | | |
|--------------------|---------------------|----------------|-----------------------|
| 1. 23,000,000,000 | 2. 54,000,000 | 3. 0.00000345 | 4. 894,000,000,000 |
| 5. 0.000000000587 | 6. 0.00000000000234 | 7. 600,000,000 | 8. 0.000000487 |
| 9. 321,000,000,000 | 10. 0.0000234 | 11. 0.00000007 | 12. 5,000,000,000,000 |

Write in decimal notation.

- | | | | |
|--------------------------|--------------------------|---------------------------|--------------------------|
| 13. 6.3×10^5 | 14. 1.2×10^{-4} | 15. 1.36×10^{-6} | 16. 6.5×10^6 |
| 17. 5.3×10^{-7} | 18. 8.4×10^{-3} | 19. 4.31×10^9 | 20. 3.2×10^{-8} |
| 21. 7×10^{10} | 22. 5×10^{10} | 23. 1×10^{-6} | 24. 9×10^8 |

Perform the operations. Leave your answer in scientific notation.

25. $(3.2 \times 10^{-72}) \cdot (1.8 \times 10^{27})$

26. $(6.2 \times 10^{42}) \div (3.1 \times 10^{-38})$

27. $(1.6 \times 10^{72}) \div (3.8 \times 10^{-83})$

28. $(4.2 \times 10^{62}) \cdot (1.2 \times 10^{-17})$

29. $(32,000,000,000,000) \cdot (0.000000000000018)$

30. $(121,000,000,000,000,000) \div (0.00000000000000011)$

31. $(0.000000000005)^2$

32. $(21,000,000,000,000)^2$

33. $\frac{(0.0018)(1400)}{(60,000)(0.07)}$

34. $\frac{(50)(0.00009)}{(0.00005)(1000)}$

35. $\frac{(800,000)(0.00003)}{(0.00004)(3000)}$

36. $\frac{(0.00002)(110000)}{22,000,000}$

37. $\frac{(4,000,000)(0.00000015)}{0.000000000003}$

38. $\frac{(81,000)(1,000,000)}{(6,000,000,000)(3,000,000)}$

39. $\frac{(78,000,000)(840,000,000)}{0.00000000000021}$

40. $\frac{(0.00000002)(36,000)}{0.000000006}$

41. $\frac{(0.00000012)^2}{(2,000,000,000)^3}$

42. $\frac{(2,000,000,000)^2}{(5,000,000,000)^3}$

43. Last year, there were about 102.5 million houses in the America. The average income of these houses is \$49,700. What is the total income of all of these houses?

44. There are about 6.02×10^{23} molecules in a mole of hydrogen. How many molecules are in 200 moles of hydrogen?

45. A terabyte holds about one trillion bytes of information. How many bytes can 2000 terabytes hold?

46. Amazon.com sells \$45 million in products a day. How much do they sell in a month of 30 days?

47. In the United States, 1.825 billion gallons of water is used every year. How much is used in a day?

48. The St. John's River discharges 2,100,000 gallons of water a week. How much does it discharge in a day?

49. A strand of DNA is about 1.5 meters long and 1.3×10^{-10} cm wide. How many times longer is DNA than it is wide?