2.3 Applications of Linear Equations

Now that we can solve equations, what are they good for?

It turns out, equations are used in a wide variety of ways in the real world. We will be taking a look at numerous applications as we move forward through the text.

In this chapter, we will take a look at just a few different types of applications.

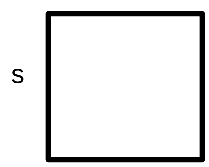
To do these applications, we will need some basic facts first.

First of all, one of the main types of applications is involving geometric figures. To do these problems involving geometry, we will need some of the following formulas.

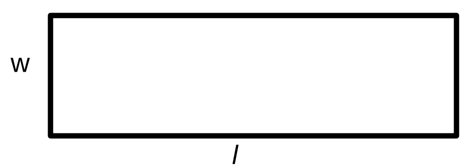
Area and Perimeter Formulas

For the following the P is the perimeter and A is the area

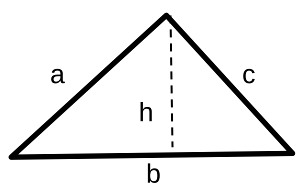
Square:
$P = 4s$
$A = s^2$



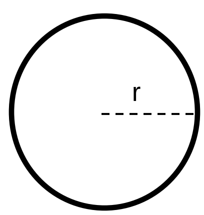
Rectangle:
$P = 2l + 2w$
$A = l \cdot w$



Triangle:
$P = a + b + c$
$A = \frac{1}{2} \cdot b \cdot h$



Circle:
$C = 2\pi \cdot r$
$A = \pi \cdot r^2$



We will also need the following fact about triangles

Angles of a Triangle

The interior angles of a triangle always add up to 180^0.

We won't need all of these formulas immediately, however, we have stated them here for use in the remainder of the textbook.

Let's take a look at a few geometry related problems in our first three examples

Example 1:

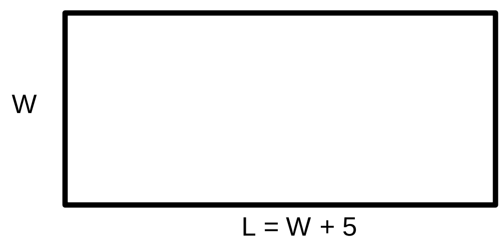
The length of a rectangle is 5 feet more than the width. If the perimeter is 30 feet, what are the dimensions of the rectangle?

Solution:

The first thing we should always do when working with a geometric figure is try to draw the figure. Notice that we are told that the length of the rectangle is 5 feet more than the width. Recall that the phrase “more than” means addition.

So if L is the length and W is the width, the length 5 feet more than the width would mean L = W + 5.

Putting this together we have the following figure



Now, we have been given the perimeter is 30 feet. Since we know $P = 2l + 2w$, if we insert everything we now know, we get

$$P = 2l + 2w$$

$$30 = 2(W + 5) + 2W$$

Once we have the equation, all we need to do is solve it for the variable, W.

$30 = 2(W + 5) + 2W$	Distribute the 2
$30 = 2W + 10 + 2W$	Combine like terms
$30 = 4W + 10$ $-10 \qquad -10$ $20 = 4W$	Move the 10 over
$\frac{20}{4} = \frac{4W}{4}$	Divide by 4 to get W alone
$W = 5$	

So our width is 5 feet. Since the length is given as L = W + 5, putting the 5 in for W we get the length is 10 feet.

Therefore, the dimensions of the rectangle are 5 feet by 10 feet.

Example 2:

The length of a rectangle is three less than two times the width. If the perimeter is 18 feet, what are the dimensions of the rectangle?

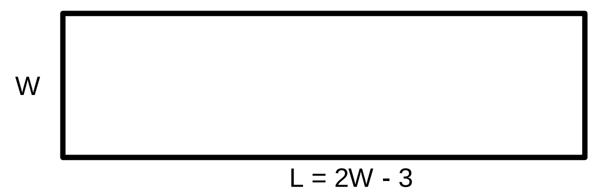
Solution:

Again we want to start by getting an idea of what our rectangle should look like. First, notice that the length is “three less than two times the width”. Recall that the phrase “less than” means to subtract, but in the back. So, “three less than two times the width” would mean

L = 2W - 3

Length two times three less than

So we get a picture of



As we did in example 1, now we use the formula for perimeter. Since we have been given that the perimeter is 18 feet, we get

$P = 2l + 2w$	Substituting the expression for L
$18 = 2(2W - 3) + 2W$	Distribute the 2
$18 = 4W - 6 + 2W$	Combine like terms
$18 = 6W - 6$ $+6 \qquad +6$	Add 6
$24 = 6W$	
$\frac{24}{6} = \frac{6w}{6}$	Divide by 6 to get W alone
$W = 4$	

So the width is 4 feet. Since the length is L = 2W - 3, if we insert the 4 for the W we get

L = 2(4) - 3

L = 8 - 3

L = 5

So the length is 5 feet. Therefore, our rectangle is 4 feet by 5 feet.

Example 3:

The second angle in a triangle is four times the first and the third angle is 30° less than the first. Find the measure of each angle.

Solution:

This time we are working on angles of a triangle. Recall, the angles of a triangle have to always add up to 180^0. With this in mind, lets call the first angle x.

Since the second angle is "four times the first", the second angle must be 4x.

Also, since the third angle is "30^0 less than the first", the third angle must be x - 30.

So, since these three angles must add up to 180, we get the equation

$$x + 4x + x - 30 = 180 \quad \text{Combine like terms}$$

$$\begin{aligned} 6x - 30 &= 180 \quad \text{Move the 30 to the other side} \\ +30 \quad & \quad +30 \end{aligned}$$

$$6x = 210$$

$$\frac{6x}{6} = \frac{210}{6} \quad \text{Divide by 6 to get x alone}$$

$$x = 35$$

So the first angle is 35^0. Since the second angle is 4 times the first, the second angle is $4(35) = 140^0$, and since the third is 30^0 less than the first, the third must be $35 - 30 = 5^0$.

Therefore, our angles are 5^0, 35^0 and 140^0.

Another type of problem we want to look at in this first look at applications is a problem involving consecutive numbers or consecutive even or odd numbers. For these, we will need the following

Consecutive Numbers

- If n is the first number, then the next consecutive numbers are: n+1, n+2, n+3, etc

Consecutive Even/Odd Numbers

- If n is the first even number, then the next consecutive even numbers are: n+2, n+4, n+6, etc.
- If n is the first odd number, then the next consecutive odd numbers are: n+2, n+4, n+6, etc.

Notice that the consecutive even and odds are done the same. The reason for this is if we assume the first number we are working with is even, the next evens are an even number of spaces away from the first even, and if we assume the first number is odd, the next odd are also an even number away from the first odd.

It may sound confusing at first, however, if you look at the odds and the evens you will see that odds are all two number apart from each other and so are the evens.

Therefore, the odds and evens should be done the same, as long as you assume the first number you are dealing with is the type of number you want to get in the end, even or odd.

Let's see some examples of consecutive number problems.

Example 4:

The sum of three consecutive integers is 63. Find the integers.

Solution:

According to what we said above, we start by saying the first consecutive number is "n". This means the next consecutive numbers are "n + 1" and "n + 2".

So, remember that "sum" means to add. Therefore, the sum of our three consecutive numbers equaling 63 means

$$n + (n + 1) + (n + 2) = 63$$

Now solve by combining like terms

$$\begin{aligned} 3n + 3 &= 63 \\ -3 \quad &- 3 \end{aligned}$$

Subtract 3 to the other side

$$3n = 60$$

$$\frac{3n}{3} = \frac{60}{3}$$

Divide to get n alone

$$n = 20$$

So, 20 is our first of three consecutive numbers. Remember, consecutive means "in order".

This means the numbers are 20, 21 and 22.

Example 5:

The sum of two consecutive odd integers is 176. Find the integers.

Solution:

Again, according to what we saw above, consecutive odd and even done the same way. So, in this case, we start by assuming that our first number "n" is odd.

If this is the case, the next consecutive odd number would be "n + 2" since each odd number is 2 numbers away from the next odd number (like 14 and 16, for example, are 2 numbers away from each other).

Since their sum is 176, we must have the equation

$$n + (n + 2) = 176$$

Combine like terms

$$\begin{aligned} 2n + 2 &= 176 \\ -2 \quad &- 2 \end{aligned}$$

Subtract 2 from both sides

$$2n = 174$$

$$\frac{2n}{2} = \frac{174}{2}$$

Divide by 2 to get n alone

$$n = 87$$

So our consecutive odd numbers must be 87 and 89.

Besides these types of problems, we can also use equations to describe a great deal of other real world situations.

In these situations, sometimes we need to generate our own equation, and sometimes we are given a formula to use.

The next two examples will illustrate each type.

Example 6:

A technical hotline charges a customer \$8 plus \$0.40 per minute to answer questions about software. How many minutes was Nate on the hotline if he was charged \$22?

Solution:

In this problem, we need to start by creating an equation that we will then solve. To begin with, let's assign a variable to what we are looking to find.

Since we want to know how many minutes Nate was on the hotline, let's call that value "x".

The hotline charges \$8 plus \$0.40 per minute. That means, there is a flat charge of \$8 just for connecting with the hotline, then a minute by minute charge for however long someone is on the phone.

This means, the amount of Nate's charge is 8 + 0.40x, since he is on for, what we have called, x minutes.

Since Nate was charged \$22, we get the equation

$$8 + 0.4x = 22$$

Now we solve as usual.

$$\begin{aligned} 8 + 0.4x &= 22 \\ -8 \qquad &\;\; -8 \end{aligned}$$ Subtract 8 from both sides

$$0.4x = 14$$

$$\frac{0.4x}{0.4} = \frac{14}{0.4}$$ Divide by 0.4 to get x alone

$$x = 35$$

Therefore, Nate was on the hotline for 35 minutes.

Example 7:

Find the time required for a falling object to increase in velocity from 16 ft/sec to 144 ft/sec. Use the formula $V = V_0 + 32t$, where V is the final velocity of a falling object, V_0 is the starting velocity of a falling object, and t is the time for the object to fall.

Solution:

In this last example, we have been given a formula to describe the situation. All we have to do in a problem like this is determine what given values to put in the various places in the formula, then solve.

Notice that V is the final velocity, and V_0 is the initial velocity. Since the object starts at 16 ft/sec and ends at 144 ft/sec, V must equal 144 and V_0 must equal 16.

Putting these into the formula gives us

$$V = V_0 + 32t$$

$$\begin{array}{l} 144 = 16 + 32t \\ -16 \ \ -16 \end{array} \qquad \text{Subtract 16}$$

$$128 = 32t$$

$$\frac{128}{32} = \frac{32t}{32} \qquad \text{Divide by 32 to get t alone}$$

$$t = 4$$

So, it takes 4 seconds to increase from 16 ft/sec to 144 ft/sec.

2.3 Exercises

1. The perimeter of a rug is 36 feet. The length of the rug is 2 feet longer than the width. Find the dimensions.

2. The length of a rectangular flower bed is 6 feet more than its width. The perimeter of the flower bed is 32 feet. Find the dimensions of the flower bed.

3. The perimeter of a rectangle is 62 cm. The length of the rectangle is 5 cm less than twice the width. Find the dimensions of the rectangle.

4. The width of a rectangle is 1 inch more than twice its length. If the perimeter is 104 inches, find the length and width of the rectangle.

5. The length of a rectangular mailing label is 2 cm less than twice the width. The perimeter is 50 cm. Find the dimensions of the label.

6. The length of a rectangular pad of paper is twice as long as the width. If the perimeter of the pad of paper is 102 cm, what are the dimensions of the pad of paper?

7. The perimeter of a college basketball court is 108 meters. The length is twice as long as the width. What is the length and width?

8. A square has perimeter of 12 yds. What is the length of the side of the square?

9. The perimeter of a rectangle is 48 inches. The length of the rectangle is 9 inches shorter than twice its width. If a square has a side that is as long as the length of this rectangle, what is the perimeter of the square?

10. The width of a rectangular pool table is 3 feet less than its length. The perimeter of the pool table is 26 feet. Find the dimensions of the pool table.

11. The perimeter of a square is 56 inches. What is the length of the side of the square?

12. The perimeter of a rectangle is 108 inches. The length of the rectangle is 6 inches more that the width. What is the area of the rectangle?

13. A triangle that has all equal sides is called an equilateral triangle. If an equilateral triangle has perimeter of 93 feet, what is the length of the side of the triangle?

14. An equilateral triangle has perimeter of 60 meters. What is the length of the side of the triangle?

15. One side of a triangle is 5 inches longer than the second side. The third side is 7 inches longer than the second side. If the perimeter is 36 inches, find the lengths of each side of the triangle.

16. A rectangle has perimeter of 50 inches. The length of the rectangle is 8 inches less than twice the width. If a square's side is as long as the width of this rectangle, what is the perimeter of the square?

17. One angle in a triangle is twice the first. The third angle is three times the first. What are the measures of the angles?

18. The second angle in a triangle is 10^0 more than the first. The third angle is 10^0 more than the second angle. Find the measure of each angle.

19. One angle in a triangle is 15^0 more than the smallest angle in the triangle. The other angle is 30^0 more than the smallest angle. Find the measures of all of the angles.

20. Two angles in a triangle have the same measure. The third angle is twice one of the others. Find the angles.

21. The sum of three consecutive integers is 42. Find the integers.

22. The sum of four consecutive integers is 230. Find the integers.

23. The sum of three consecutive odd integers is 51. Find the integers.

24. The sum of three consecutive even integers is 186. Find the integers.

25. The smallest of three consecutive integers is 24 less than the sum of the two larger integers. Find the three integers.

26. The smallest of three consecutive integers is 26 less than the sum of the two larger integers. Find the integers.

27. John "Crazy Plumber" Vadnais charged $445 for a water softener and instillation. The charge included $310 for the water softener and $45 per hour for labor. How many hours were required to install the water softener?

28. *Fly by Night Electric* charged $1775 for a rewire job. This charge included $180 for parts and $55 per hour for labor. How many hours did the rewire job take?

29. Tristan charges you $278 for performing a 30,000-mile checkup on your car: This charge includes $152 for parts and $42 per hour for labor. Find the number of hours Tristan worked on your car.

30. Dan's Cement sells cement for $75 plus $4 per yard. How many yards of cement can be purchased for #363?

31. A water flow restrictor has reduced the flow of water to 2 L/min. This amount is 1 L/min less than three fifths the original flow rate. Find the original flow rate.

32. Jim charges you $338 to fully detail your car. This charge includes $152 for supplies and $62 per hour for labor. How many hours did Jim take to detail your car?

33. Mark charged $492 to build a flower bed in Jennifer's back yard. This cost included $100 for materials and $24.50 per hour for labor. How many hours did it take Mark to build the flower bed?

34. A roofer charged $115 to repair a small leak. This charge included $40 for materials and $30 per hour for labor. How long did it take to repair the leak?

35. A bricklayers union charges monthly dues of $3 plus $.18 for each hour worked during the month. A union member's dues for July were $31.80. How many hours were worked in July?

36. The value of a vintage car this year is $42,000. This is five fourths the value two years ago. What was the value two years ago?

37. Four hundred seventeen men have flown into space. This number is 25 more than 8 times the number of women who have flown into space. How many women have flown into space?

38. Marty earned $3600 last month. This salary is the sum of his base salary of $1200 and a 6% commission on total sales. What were Marty's total sales last month?

39. A sales executive receives a base monthly salary of $600 plus a 8.25% commission on total sales per month. Find the executives total sales during a month in which he had total compensation of $4109.55.

40. The state income tax for Eric last month was $256. This amount is $5 more that 8% of his monthly salary. What is Eric's monthly salary?

41. The monthly income for a manager of an apartment complex was $3500. This includes the manager's base salary of $2500 plus a 1% bonus on total sales. Find the managers total sales for the month

42. Andy harvested 28,336 bins of peaches this year. This amount is a 12% increase over last year's crop. How many bins did Andy harvest last year?

43. The formula for distance traveled *D* on *G* gallons of gas with a car traveling *M* miles per gallon is given by $D = M \cdot G$. Sue averaged 28 mi/gal on a 621 mile trip to LA and back. How many gallons of gas did Sue use?

44. The formula for distance traveled *D* on *G* gallons of gas with a car traveling *M* miles per gallon is given by $D = M \cdot G$. Tom averaged 32 mi/gal on a 592 mile trip. How many gallons of gas did Tom use?

45. Find the time required for a falling object to increase in velocity from 24 ft/sec to 392 ft/sec. Use the formula $V = V_0 + 32t$, where V is the final velocity of a falling object, V_0 is the starting velocity of a falling object, and t is the time for the object to fall.

46. Find the time required for a falling object to increase in velocity from 8 ft/sec to 472 ft/sec. Use the formula $V = V_0 + 32t$, where V is the final velocity of a falling object, V_0 is the starting velocity of a falling object, and t is the time for the object to fall

47. The relationship between Celsius and Fahrenheit temperature is given by the formula $C = \frac{5}{9}(F - 32)$, where C is the Celsius temperature and F is the Fahrenheit temperature. Find the Fahrenheit temperature when the Celsius temperature is -40^0.

48. The relationship between Celsius and Fahrenheit temperature is given by the formula $C = \frac{5}{9}(F - 32)$, where C is the Celsius temperature and F is the Fahrenheit temperature. Find the Fahrenheit temperature when the Celsius temperature is 100^0.

49. The formula $T = U \cdot N + F$ represents the total cost of a product which is being produced where *T* is the total cost, *U* is the cost per unit, *N* is the number of units produced and *F* is the fixed costs. Find the number of units made during a week in which the total cost was $25,000, the cost per unit was $8 and the fixed costs were $5,000.

50. The formula $T = U \cdot N + F$ represents the total cost of a product which is being produced where *T* is the total cost, *U* is the cost per unit, *N* is the number of units produced and *F* is the fixed costs. Find the cost per unit during a week in which the total cost was $80,000, the number of units produced was 500 and the fixed costs were $15,000.

51. The formula $T = I \cdot R + B$ represents the monthly tax on income where *T* is the monthly tax, *I* is the monthly income, *R* is the tax rate, and *B* is the base monthly tax. Ethan pays $476 in tax this month. His monthly tax rate is 22% and the base monthly tax is $80. What is Ethan's monthly income?

52. The formula $T = I \cdot R + B$ represents the monthly tax on income where *T* is the monthly tax, *I* is the monthly income, *R* is the tax rate, and *B* is the base monthly tax. Victoria pays $770 in tax this month. Her monthly income is $3100 and the base monthly tax is $150. What is Victoria's tax rate?

53. The formula for selling price of a product sold at a store is given by $S = C + r \cdot C$, where *S* is the selling price, *C* is the cost and *r* is the markup rate. A bike shop sells a mountain bike for $232.50. The cost to the store is $187.50. What markup rate does the bike shop use?

54. The formula for selling price of a product sold at a store is given by $S = C + r \cdot C$, where *S* is the selling price, *C* is the cost and *r* is the markup rate. A toy store uses a markup of 30% on all boxes of Lego's. If the store sells a large box of assorted Legos for $18.85, what is the cost to the store?