2.2 General Equations

Now that we can solve the basic equations as we learned in the last section, we need to turn our attention to solving more complex equations.

It's important to note that the equations we are solving in this chapter all have a power of x that is no bigger than the first power. We refer to equations of this sort as **linear equations**.

We have a clearly defined process for solving linear equations. It is given below.

Solving Linear Equations

1. Simplify each side of the equation:
   Remove grouping symbols, clear fractions, combine like terms, etc.
2. Use the add/subtract property to get all variable terms on one side, and constant terms on the other side.
3. Use the multiply/divide property to get the equation to say "variable=number".
4. Check.

Keep in mind that our main goal is still the same… we want the equation to say "variable = number". Let's begin by solving a few simpler equations just to get ourselves going.

Example 1:

Solve.

a. $2x + 2 = 12$ b. $2(2x - 1) = 6$ c. $2x - (3 - x) = 0$

Solution:

a. First, we solve this as we did in the last section. That is, subtract the 2 from both sides, then divide by 2. We get

$$2x + 2 = 12$$
$$-2 \quad -2$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

So the solution set is {5}.

b. Here, we need to start by simplifying the side, by distributing the 2 through. Then we proceed as usual. We have

$$2(2x - 1) = 6$$ Distribute the 2

$$4x - 2 = 6$$ Add 2 to both sides
$$+2 \quad +2$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$ Divide by 4

$$x = 2$$

So the solution set is {2}.

c. Again, here, we need to start by simplifying the side. Also, just like in part b above, we need to distribute to get rid of the parenthesis.
We have

$2x-(3-x)=0$	Distribute -1
$2x-3+x=0$	Combine like terms
$3x-3=0$ $+3 \quad +3$	Add 3 to both sides
$3x=3$	
$\frac{3x}{3}=\frac{3}{3}$	Divide by 3
$x=1$	

So the solution set is {1}.

Now, we need to look at solving some much more complicated equations. The process remains the same as above, we just have to be careful to take it one step at a time and work our way through each equation.

Example 2:

Solve.

a. $6(3x-2)=7(x-3)-2$ b. $3(7-2x)=30-7(x+1)$

c. $5+3[1+2(2x-3)]=6(x+5)$

Solution:

a. We again follow the steps outlined above. We begin by simplifying each side as much as possible. Meaning, in this case, distribute and combine like terms. Then we continue as before.
We get

$6(3x-2)=7(x-3)-2$	Distribute the 6 and 7
$18x-12=7x-21-2$	Combine like terms
$18x-12=7x-23$ $-7x \quad\quad -7x$	Get the variable terms on one side
$11x-12=-23$ $+12 \quad +12$	Get the constants on the other side
$11x=-11$	
$\frac{11x}{11}=\frac{-11}{11}$	Divide by 11
$x=-1$	

So our solution set is {-1}.

b. Again, we start by simplifying each side as seen below.

$$3(7-2x)=30-7(x+1)$$ Distribute 3 and 7

$$21-6x=30-7x-7$$ Combine like terms

$$\begin{aligned}21-6x&=23-7x\\ +7x\quad&\quad\ +7x\end{aligned}$$ Get the variables on one side

$$\begin{aligned}21+x&=23\\ -21\quad&\quad -21\end{aligned}$$ Get the constants on the other

$$x=2$$

So the solution set is {2}.

c. Lastly, we solve as we did above. Start by simplifying each side. Notice here, however, that the left side is quite complicated. We need to start working from the inside out in order to simplify this side. It looks as follows

$$5+3[1+2(2x-3)]=6(x+5)$$ Distribute inside the [ ] on the left and the 6 on the right

$$5+3[1+4x-6]=6x+30$$ Combine like terms inside the [ ]

$$5+3[4x-5]=6x+30$$ Distribute the 3

$$5+12x-15=6x+30$$ Combine like terms

$$\begin{aligned}12x-10&=6x+30\\ -6x\quad&\quad -6x\end{aligned}$$ Get the varibles on one side

$$\begin{aligned}6x-10&=30\\ +10\quad&\quad +10\end{aligned}$$ Get the constants on the other

$$6x=40$$

Divide by 6

$$\frac{6x}{6}=\frac{40}{6}$$

Reduce

$$x=\frac{20}{3}$$

So the solution set is $\left\{\frac{20}{3}\right\}$.

Finally, we need to look at dealing with equations that contain fractions or decimals.

<u>Example 3</u>:

Solve.

a. $\frac{7}{8}x-\frac{1}{4}+\frac{3}{4}x=\frac{1}{16}+x$

b. $\frac{1}{6}\left(\frac{3}{4}x-2\right)=-\frac{1}{5}$

c. $0.13x+0.17(100-x)=14$

Solution:

a. To deal with any equation containing a fraction, we need to remember that the first step in solving an equation is to simplify each side. Part of simplifying each side is "clearing fractions" as we learned in the last section.

Recall that to clear fractions, we multiply each side by the LCD. In this case the LCD is 16. Once we multiply each side by 16, the fractions should disappear and we should be able to continue as normal.

$$\frac{7}{8}x-\frac{1}{4}+\frac{3}{4}x=\frac{1}{16}+x$$

Multiply by the LCD, 16

$$16\cdot\left(\frac{7}{8}x-\frac{1}{4}+\frac{3}{4}x\right)=\left(\frac{1}{16}+x\right)\cdot 16$$

Distribute 16 to each term

$$16\cdot\frac{7}{8}x-16\cdot\frac{1}{4}+16\cdot\frac{3}{4}x=16\cdot\frac{1}{16}+16\cdot x$$

Simplify the fractions

$$14x-4+12x=1+16x$$

Combine like terms

$$\begin{aligned}26x-4&=1+16x\\ -16x\qquad &\quad\ -16x\end{aligned}$$

Get the variables on one side

$$\begin{aligned}10x-4&=1\\ +4\quad&\ +4\end{aligned}$$

Get the constants on the other

$$10x=5$$

Divide by 10

$$\frac{10x}{10}=\frac{5}{10}$$

$$x=\frac{1}{2}$$

So the solution set is $\left\{\frac{1}{2}\right\}$.

b. Again, here, we will need to simplify each side first, which means we will need to clear the fractions.

However, as it turns out, in this situation, it's always best to clear out the parenthesis first, then clear the fractions.

In general, its best to simplify in the order: remove parenthesis, clear fractions, then combine like terms. The reason for this is by taking the operations in this order, the chances of silly errors will greatly decrease.

So proceeding as we said, we get

$$\frac{1}{6}\left(\frac{3}{4}x-2\right)=-\frac{1}{5}$$

Distribute to clear parenthesis

$$\frac{1}{8}x-\frac{1}{3}=-\frac{1}{5}$$

Clear fraction by multiplying by the LCD of 120

$$120\cdot\left(\frac{1}{8}x-\frac{1}{3}\right)=\left(-\frac{1}{5}\right)\cdot 120$$

$$\begin{aligned}15x-40&=-24\\ +40&\quad +40\end{aligned}$$

Get the variables on one side and constants on the other

$$15x = 16$$

$$\frac{15x}{15} = \frac{16}{15}$$

$$x = \frac{16}{15}$$

So the solution set is $\left\{\frac{16}{15}\right\}$.

c. Here we are solving an equation containing decimals. The best way to deal with an equation containing decimals is to not be bothered by the decimals. Treat them the same way as you would any old numbers.

With this in mind, we solve as normal.

$$0.13x + 0.17(100 - x) = 14$$ Distribute the 0.17

$$0.13x + 17 - 0.17x = 14$$ Combine like terms

$$-0.04x + 17 = 14$$ Get the 17 to the other side

$$-17 \quad -17$$

$$-0.04x = -3$$ Divide by -0.04

$$\frac{-0.04x}{-0.04} = \frac{-3}{-0.04}$$

$$x = 75$$

So the solution set is {75}.

In part c of the last example, we should note that there is another way to solve an equation containing decimals. We can also "clear decimals" by multiplying both sides of the equation by whatever power of 10 will eliminate the decimals (that is 10, 100, 1000, etc).

If we do this, the equation works out to exactly the same solution. Either method is acceptable for solving an equation containing decimals.

2.2 Exercises

Solve.

1. $-2x + 5 = -9$
2. $5x + 4 = 13$
3. $-9x + 4 = -14$
4. $4x - 3 = 7x + 2$
5. $8 - 3x = 2x - 8$
6. $8x - 3 = 4 - x$
7. $8x - 3(x - 2) = 12$
8. $-3(x - 2) - 5 = 1$
9. $-4(x + 2) = 0$
10. $2(x - 2) = 0$
11. $5 - (2x - 4) = 15$
12. $1 - 6(2x - 1) = 2$
13. $12(x + 3) = 7(x + 3)$
14. $3(x - 3) = 12(x - 3)$
15. $2x - 3(2x - 5) = 7 - 2x$
16. $4 - (x - 15) = 4 + x$
17. $6(3 - x) = 5 - (4x - 7)$
18. $3(x - 2) = 7 - 2(x - 7)$

19. $2(3-2x)=4-(3x-6)$

20. $2(1-2x)=3-(x+5)$

21. $6-(2x-1)=5(3x-4)$

22. $3-(13-2x)=5(x-2)$

23. $2(x+7)-9=5(x-4)$

24. $6(x-3)-1=2(x+3)$

25. $5-3(2x-3)=2x-10$

26. $4(2-2x)=5+2(3x+1)$

27. $-2(4-3x)+1=-2(x-6)$

28. $-(x+3)+1=5-(4x-8)$

29. $2-2(4-x)=2x-3(2x-4)$

30. $2-3(3-x)=5x-(4x+1)$

31. $2(3-4x)+4=2-3(4-4x)$

32. $6(x-1)+2x=5-3(x-1)$

33. $3x-10(x-1)=2(3x+4)-11$

34. $2x-(3x+2)=4x-(3x+2)$

35. $4(3x-2)=2[3x-(2x-1)]$

36. $4(3x-2)=2[3x-(2x-1)]$

37. $3[2-5(x+1)]=-4(2x+1)$

38. $2[3-(3x+1)]=3(x+1)$

39. $6[x-(5x-7)]=4-5x$

40. $3[x-2(3x+7)]=-(x+1)$

41. $-4[1+3(x-1)]=-2(x+6)$

42. $2[1-(6x-1)]=-3(x-3)$

43. $4[1-3(x+1)]=5(x+2)$

44. $3[1-(x+1)]=x-(2x+1)$

45. $3(2x-1)-(5x-4)-(7x+1)=-3$

46. $2(3x-2)-2(x-8)-(6x+1)=0$

47. $3(2x-1)-(4x+1)-2(5x-6)=8$

48. $2(12x+6)-(x-3)-7(x-2)=7$

49. $5(x-2)-2(4x-3)-(x+4)=-8$

50. $4(x+5)-2(4x+4)-(3x+1)=11$

51. $3(4x-7)-2(x+8)-(3x+1)=4-2(x-1)$

52. $2(x+7)-(2x+5)-(5x-1)=6(x+7)-2(3x+11)$

53. $\frac{1}{3}x+1=\frac{1}{12}x-4$

54. $\frac{11}{6}x+\frac{1}{3}=0$

55. $1-\frac{2}{3}x=\frac{9}{5}-\frac{1}{5}x+\frac{3}{5}$

56. $\frac{1}{4}-\frac{2}{7}x=-\frac{1}{5}x+\frac{3}{8}$

57. $\frac{1}{14}(x+4)=\frac{2}{7}$

58. $\frac{1}{8}(2x-4)=\frac{1}{3}$

59. $\frac{2}{3}\left(\frac{7}{8}-4x\right)-\frac{5}{8}=\frac{3}{8}$

60. $\frac{1}{5}\left(\frac{7}{2}-x\right)+\frac{5}{2}=\frac{1}{3}$

61. $\frac{1}{3}(25-4x)=\frac{1}{4}(5x+12)+6$

62. $\frac{1}{6}\left(18-\frac{2}{3}x\right)=3x-\frac{1}{4}\left(x+\frac{1}{2}\right)$

63. $0.42x+0.66(120-x)=0.58(120)$

64. $0.2x+0.16(10-x)=1.8$

65. $0.14x+0.07(7000-x)=581$

66. $0.52x+0.65(1000-x)=0.58(1000)$

67. $0.15x+0.05(x+4000)=1600$

68. $0.29x+0.27(300+x)=84$