1.5 Set Notation

In this section we take an aside from the normal discussion in algebra.

We want to take a look at the topic of sets and set notation. The reason we want to do this is so that as we encounter various types of sets throughout the remainder of the text, we will be able to deal with them properly instead of just mentioning them and moving on.

We begin with the following definition.

Definition: **Set**- a collection of objects written in braces. Each object in the set is called an **element** or **member** of the set.

Let's get an example to see exactly what these "sets" look like.

Example 1:

Write the following in set notation.

a. The set of the first 3 letters of the alphabet

b. The set of even integers between -11 and -1

Solution:

a. A set is simply just a list of the elements that are described, written in braces. So in this case, the set must be

$$\{a, b, c\}$$

b. This time, the set is just the numbers between -11 and -1. So we must have

$$\{-10, -8, -6, -4, -2\}$$

The method of writing a set by listing its members is called the **roster method**. We use the roster method for many of the sets we deal with, however, we will also need to deal with other methods of writing sets later in this section.

Next, we have a very special type of set.

Definition: **Empty Set**- the set that contains no members, written { } or ∅.

We will sometime run across a situation in which no values are in the set. Here is an example of one such situation.

Example 2:

Write set of all integers between ½ and ¼ using the roster method.

Solution:

Since there are no integer values that lie between ½ and ¼ this set contains no elements. Therefore we have the empty set, ∅.

As we have seen in the past, and as we will continue to see for a while, once we define something, we are generally interested in doing operations on what we have defined.

Since sets are very different than other things we study, the operations are a little different.

There are quite a few set operations that we could discuss, however, we will only take a look at the ones which are useful for our discussions throughout this text, namely, the operations of union and intersection.

Definition:
Union- written $A \cup B$- the set of all elements which are in either set A or in set B
Intersection- written $A \cap B$- the set of all elements which common to set A and set B

Union and intersection are quite easy to deal with.

All we need to know is that the union is like the large melting pot where we dump all of the elements of all of the sets involved.

On the other hand, the intersection is only the elements that the sets have in common.

Let's see an example.

Example 3:

Find $A \cup B$ and $A \cap B$.

a. $A = \{-3, -2, -1\}, B = \{-2, -1, 0\}$

b. $A = \{a, b, c\}, B = \{x, y, z\}$

c. $A = \{cats\ with\ black\ hair\}, B = \{cats\ with\ white\ hair\}$

Solution:

a. First we will start with the union. The union is the set that contains all of the elements in BOTH sets. Meaning, if something is in set A, it is in the union. If something is in set B, then it is also in the union.

   The only thing we must be aware of is that we never repeat an element of a set. It is sufficient for the element to be in the set, it doesn't need to show up more than once.

   With this in mind, we have a union of
   $$A \cup B = \{-3, -2, -1, 0\}$$

   Now, for the intersection, we only want the elements that are common to both sets. So in this case, the only common elements are -2 and -1. So the intersection is

   $$A \cap B = \{-2, -1\}$$

b. Again, for the union we take all of the elements in both sets. We get

   $$A \cup B = \{a, b, c, x, y, z\}$$

   For the intersection we get just what is in common. Since the sets have nothing in common, the intersection must be empty.

   $$A \cap B = \emptyset$$

c. Lastly, these sets are a little more descriptive and this sometimes makes them a little harder to work with. First, the union is all of the elements in both sets. So in this case, we want all of the "cats that have black hair" as well as all of the "cats that have white hair" in the same set.

   We can describe it as

$$A \cup B = \{cats\ that\ have\ black\ hair\ or\ white\ hair\}$$

   For the intersection, we want to describe the just the cats who have both black and white hair. So we would say just that

$$A \cap B = \{cats\ that\ have\ black\ and\ white\ hair\}$$

In part c of the last example, we run into a very important thing to understand about the union and intersection. The key to these sets is that the union is usually associated with the word "or", and the intersection is usually associated with the word "and".

This will be important in a coming chapter.

The last thing we want to do with sets is define another way of writing a set. Up to this point we have only used the roster method to write a set. However, some sets are too large to write by listing out its members.

For this reason, we have another notation.

Set-Builder Notation

Mathematics is filled with unusual and new symbols. To do set builder notation we need a couple of these symbols:

ϵ – is a member of | - such that

Set-builder notation is best seen by looking at some examples. Let's take a look.

Example 4:

Write in set-builder notation.

a. The set of all positive integers less than 20

b. The set of odd integers greater than -2

c. The set of real numbers less than 57

Solution:

a. The way set-builder notation works is we start by defining a variable that we use to generate the elements of the set, then we use the "such that" and "is a member of" symbols to describe those elements, in terms of x.

   So, here is how it goes. Every set in set-builder notation begins with

$$\{x| \quad \}$$

   Now, we simply need to describe the elements by using the x. Notice we have two different things we want in the set. We want "positive integers" and we want "less than 20".

First, to describe the "less than 20" part we can simply say x<20.

Now the harder part. To make this process easier, the "positive integer" part, can just be stated as $x\epsilon\ positive\ integers$. There are other ways to describe the positive integers, however, they are beyond the scope of this text.

Putting it together we get the set-builder notation of

$$\{x|x < 20, x\epsilon\ positive\ integers\}$$

b. Again, all set-builder notation sets start as

$$\{x| \quad \}$$

Now we, again have two things to describe, the "greater than -2" part, which we can state as x>-2, and the "odd integers" part, which we state as $x\epsilon\ odd\ integers$.

Placing those in the set we get

$$\{x|x > -2, x\epsilon\ odd\ integers\}$$

c. Lastly, we start our set as

$$\{x| \quad \}$$

We again have two things to describe, "less than 57", which is x<57, and the "real numbers" part.

As it turns out, whenever we want to state that we are working in the real numbers, we can leave that part off.

The reason is, the basic assumption of a set in set-builder notation is that we are working in the real numbers. So to leave it off, implies we are in the real numbers.

Therefore our set is simply

$$\{x|x < 57\}$$

1.5 Exercises

Use the roster method to write the set described.

1. the set of whole numbers between 20 and 30
2. the set of whole numbers between 6 and 15
3. the set of integers between -10 and -1
4. the set of integers between -2 and 7
5. the set of odd integers between -22 and -6
6. the set of even numbers between -103 and -99
7. the set of even integers between 11 and 15
8. the set of odd integers between 28 and 42
9. the set of integers between -1 and 0
10. the set of integers between 15 and 38
11. the set of integers after -3 but before 5
12. the set of integers between 0 and 1
13. the set of female roosters
14. the set of vowels

15. the set of letters after Q
16. the set of letters before F
17. the set of letters before L
18. the set of letters after Q
19. the set of letters between L and Q
20. the set of letters between A and M

Find $A \cup B$ and $A \cap B$.

21. $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$
22. $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$
23. $A = \{a, b, c, d, e\}$ and $B = \{c, d, e, f, g\}$
24. $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 4, 5\}$
25. $A = \{-5, -4, -3, -2, -1\}$ and $B = \{-4, -3, 0, 1, 2\}$
26. $A = \{-1, -2, -3, -4\}$ and $B = \{-2, -3, -4, -5\}$
27. $A = \{1, 4, 7, 8, 11\}$ and $B = \{1, 2, 3, 4, 5, 6\}$
28. $A = \{w, x, y, z\}$ and $B = \{a, b, c, d\}$
29. $A = \{w, x, y, z\}$ and $B = \{u, v, w, x, y\}$
30. $A = \{11, 21, 31, 41\}$ and $B = \{21, 32, 42, 51\}$
31. $A = \{males\ in\ math\ 200\}$ and $B = \{males\ in\ english\ 1\}$
32. $A = \{dogs\ with\ long\ hair\}$ and $B = \{cats\ with\ long\ hair\}$
33. $A = \{-17, -16, -12, -10\}$ and $B = \{-15, -10, -5\}$
34. $A = \{0, 1, 2, 3, 4\}$ and $B = \{-2, -1, 0, 1, 2, 3\}$
35. $A = \{-3, -1, 1\}$ and $B = \{-2, 0, 2\}$
36. $A = \{11, 12, 13, 14\}$ and $B = \{12, 13, 15\}$
37. $A = \{a, b, x, y\}$ and $B = \{a, b, c, x, y, z\}$
38. $A = \{n, m, o, p\}$ and $B = \{o, p, q, r, s, t\}$
39. $A = \{1, 2, 5, 6, 8\}$ and $B = \{-2, 3, 5, 6, 7, 8\}$
40. $A = \{-2, 3, -4\}$ and $B = \{2, 3, 4, 5\}$
41. $A = \{a, b, c, d, e, f\}$ and $B = \{g, h, i, j, k\}$
42. $A = \{a, e, i, o\}$ and $B = \{u\}$
43. $A = \{x, y, z\}$ and $B = \{x\}$
44. $A = \{j, n, b\}$ and $B = \{o\}$
45. $A = \{-5, -4, -3, -2\}$ and $B = \{-4, -2, 0, 2\}$
46. $A = \{-4, -3\}$ and $B = \{-3, -2\}$
47. $A = \{1, 3, 5\}$ and $B = \{2, 4, 5\}$
48. $A = \{2, 4\}$ and $B = \{4, 6\}$
49. $A = \{7\}$ and $B = \phi$
50. $A = \{0\}$ and $B = \phi$

Use set builder notation to write the set described.

51. the set of integers less than 40
52. the set of integers greater than 12
53. the set of real numbers greater than -12
54. the set of rational numbers greater that .3/4
55. the set of rational numbers less than -1/2
56. the set of real numbers less than -3.2
57. the set of integers between -8 and -150
58. the set of rational numbers less than 3
59. the set of real numbers between -0.75 and -0.675
60. the set of integers between -1 and 1,000
61. the set of multiples of -3
62. the set of divisors of -12,000
63. the set of numbers divisible by 7
64. the set of multiples of 7
65. the set of odd integers larger than -12
66. the set of even integers less than -47
67. the set of multiples of 3.8
68. the set of multiples of 1.4
69. the set of irrational numbers divisible by π
70. the set of real numbers between -22/7 and 1.9