1.4 Variable Expressions

Now that we can properly deal with all of our numbers and numbering systems, we need to turn our attention to actual algebra.

Algebra consists of dealing with unknown values. These unknown values we call **variables** and we use a variety of symbols and letters to represent these variables.

We spend a great deal of time working with the combination of variables, numbers and operations. We call these

> **Definition**: **Variable Expression**- a combination of basic operations and one or more variables.

The items in a variable expression which are separated by a + or - symbol are called the **terms** of a variable expression. Terms which contain variables are called **variable terms**, and term without variables are called **constant terms**.

Here is an example of a variable expression

Variable terms constant term

$$2x^2 - 4y^2 + x - 3xy - y + 6$$

6 terms

The number in front of a term is called the **coefficient**

So the coefficient of $2x^2$ is 2, and the coefficient of -3xy is -3.

Notice, if a term does not seem to have a coefficient, it must be a 1. There is always an invisible 1 in front of every term.

So, the coefficient of x = 1x is 1, and also, the coefficient of –y = -1y is -1. This also means, when a coefficient is 1, we tend not to write it.

There are tons of things we want to do with variable expressions. The first thing we want to do is to be able to evaluate a variable expression if we happen to know what the values of the variables are.

To do this, we have the following process.

Evaluating a Variable Expression
1. Substitute the value into the expression. 2. Use the order of operations to simplify.

Let's take a look at an example of evaluating a variable expression.

Example 1:

Evaluate for the given values of the variables.

a. $2a-(c+a)^2$ for $a=2, c=-4$

b. $\frac{b^2-a}{ad+3c}$ for $a=-2, b=4, c=-1, d=3$

Solution:

a. To evaluate a variable expression, we need to start by substituting each value into the expression where they belong. So in this case we have

$$2a-(c+a)^2$$
$$=2(2)-(-4+2^2)$$

Then we follow the order of operations that we learned in the last section. We get

$2(2)-(-4+2^2)$	Simplify inside grouping symbols, exponent first
$=2(2)-(-4+4)$	
$=2(2)-0$	Multiply
$=4$	

b. Again, we substitute the variable values into the expression and then follow the order of operations.

$\frac{b^2-a}{ad+3c}$	Substituting we have
$=\frac{4^2-(-2)}{(-2)(3)+3(-1)}$	Exponents on top Multiplying on bottom
$=\frac{16-(-2)}{-6+(-3)}$	Add and subtract
$=\frac{18}{-9}$	Finish by dividing
$=-2$	

As it turns out, most of the time we don't know the values of the variables. So, we need to work with then simply as variables.

To do this we need a few things. We start with

<u>Definition</u>: **<u>Like Terms</u>**- terms which have the same variable part, same variables to the same powers.

3xy and -4xy are like   2x and $3x^2$ are not like

According to this definition (specifically the note), the terms 7ab and 8ba are like terms. This will be important later in the textbook.

The most important thing we do with like terms is we combine them together as a single term.

Combining Like Terms

To combine like terms we simply add or subtract the coefficients.

Let's take a look at combining like terms in the next example.

Example 2:

Simplify.

a. $6x + 8x$ b. $5ab – 7ba$ c. $7x – 8x + 3y$

d. $8x^2y + 8xy^2 – 7x^2y$ e. $x^2 – 7x – 5x^2 + 5x$

Solution:

a. To combine like terms, we just need to add the coefficients together. So, since these terms are clearly like, we get

$$6x + 8x = 14x$$

b. As we talked about above, the order of the variables does not matter in a term. So, 5ab and 7ba are like terms. So, we combine them by subtracting coefficients.

Since the variable parts "look" different but are actually the same, we can choose either one for our answer. We have

$$5ab - 7ba = -2ab$$

c. This time, we need to start by identifying which terms are like. In this case, the 7x and the -8x are clearly like terms. The term 3y is not like and therefore will not combine with the other two.

So we subtract the coefficients of the like terms, and simply drag along the unlike terms. We get

$$7x - 8x + 3y$$

$$= -x + 3y$$ Remember when the coefficient is 1, we do not write it

d. Again, we need to start with identifying the like terms. Remember, terms are like if they have the same variables to the same powers. This means, the first term and the last term are like, but the middle on is not like, since the exponents are in different places. So we combine like usual.

$$8x^2y + 8xy^2 – 7x^2y$$

$$= x^2y + 8xy^2$$

e. Lastly, we again identify that the first and third term are like and the second and fourth term are like. Therefore, we combine them as follows.

$$x^2 – 7x – 5x^2 + 5x$$

$$= -4x^2 – 2x$$

Now that we can combine like terms to simplify a variable expression, we need to take a look at a very important, in fact, one of the most important properties in algebra.

Distributive Property

$$a(b + c) = a \cdot b + a \cdot c$$

Even though it appears that the distributive property violates the order of operations, it does not. In fact, we usually use the distributive property when there is nothing we can do inside the parenthesis anyway.

As we said, the distributive property is a vital part of algebra. If we do not master it, we will not be able to do much of anything in algebra.

Here are some examples to help us master the distributive property.

Example 3:

Simplify.

a. $-3(2y - 8)$ b. $4(-3a^2 - 5a + 7)$ c. $-(x + 2)$

Solution:

a. The idea of the distributive property is that the value on the outside of the parenthesis can be "distributed" to each term on the inside by multiplication. So, the -3 will get multiplied by each term on the inside. Remember that the sign in front of a number is attached to the number.
We have

$$-3(2y - 8)$$

$$= -3(2y) + (-3)(-8)$$

$$= -6y + 24$$

b. Here, we need to know that the distributive property works no matter how many terms are inside the grouping symbols. Therefore, we can multiply the 4 by each of the three terms inside the parenthesis. We get

$$4(-3a^2 - 5a + 7)$$

$$= 4(-3a^2) + 4(-5a) + 4(7)$$

$$= -12a^2 - 20a + 28$$

c. Lastly, it appears that there is no number to distribute on the outside of the parenthesis. However, recall that there are invisible 1's all over the place. In this case, the "-" is actually a -1. So, we can put it in, and distribute as usual.

$$-(x + 2)$$

$$= -1(x + 2)$$

$$= -x - 2$$

Now some harder examples.

Example 4:

Simplify.

a. $x - 2(x - 1)$ b. $2(5x - 2) - (10x + 4) + 8$ c. $-7x + 3[x - (3 - 2x)] - 2x$

Solution:

a. Here we need to start by using the distributive property to get rid of our parenthesis.

$$x - 2(x - 1)$$
$$= x - 2x + 2$$

Now we combine like terms

$$x - 2x + 2$$
$$= -x + 2$$

b. Again, we need to start by using the distributive property on each set of parenthesis. Then combine like terms as we did in part a.

$$2(5x - 2) - (10x + 4) + 8$$
$$= 10x - 4 - 10x - 4 + 8$$

When we combine like terms we notice that they all cancel. When this happens, the answer is 0.

c. Lastly, we have a number of grouping symbols involved. As it turns out, the distributive property works the same on parentheses as well as brackets. So, as we would do with the order of operations, we start working from the inside out, combining like terms as we go. We get

$-7x + 3[x - (3 - 2x)] - 2x$	Distribute the -1 inside
$= -7x + 3[x - 3 + 2x] - 2x$	Combine the x and 2x
$= -7x + 3[3x - 3] - 2x$	Distribute the 3
$= -7x + 9x - 9 - 2x$	Combine like terms
$= -9$	

Lastly in these beginning foundations of algebra, we want to be able to translate words into mathematical statements.

This is important because we will want to deal with a variety of applications throughout this textbook and in order to do them, we will need to be able to get the statement out of a verbal statement and into a math statement.

Below we have listed some common phrases for each of the basic operations.

Phrases for Translation

Addition:	Subtraction:
More than	Less than
The sum of	The difference between
The total of	Decreased by
Increased by	
Multiplication:	**Division:**
Times	Divided by
The product of	The quotient of
Of	The ratio of
Twice (two times)	
Thrice (three times	
Exponents:	
To the power of	
To the	
Squared (to the 2nd power)	
Cubed (to the 3rd power)	

For the most part, you take these phrases and insert the values in the order that they appear. The exceptions to that are the phrases "more than" and "less than". When you see one of these two phrases appear, you must switch the order.

So, for example, if we have "5 less than x", this translates to x - 5, or if we have "6 more than a number" we would have x + 6 (assuming our "number" is defined to be x).

With all of this in mind, let's look at the next example.

Example 5:

Translate into a variable expression.

a. The sum of a number and 12

b. five less than twice a number

c. The product of a number and two more than the number

d. The quotient of six and the sum of a number and nine

Solution:

a. To translate the phrases, we first need to establish a variable for the "number". We can use any letter we want for the variable. For the same of simplicity, lets "the number" be x.

Since we know that "sum" means adding, we have the expression

$$x + 12$$

the number sum of

b. Again, let's let "the number" be x. Remember that the phrase "less than" means subtract, but in reverse, or better yet, subtract in back. Also, since twice means 2 times, we have

$$2x - 5$$

twice the number 5 less than

c. As above, we will start with saying that "the number" is x. This time, we have "the product", which means multiply, and "more than" which means add in the back.

Since it says the product of the number and two more than the number, we have to make sure we multiply the "x" by the entire addition of x + 2. Otherwise, it wouldn't be the product of x **<u>and</u>** two more than the number. The "and" part of this statement tells us the multiplication is between the x and the rest.

So, we use some parenthesis to keep the x + 2 together. This gives us

x(x + 2)

d. Lastly, we will again let "the number" be x. Similar to part c above, we need to be careful of how this is arranged. We know "quotient" means to divide and "sum" means add, but since it is the quotient of 6 **<u>and</u>** the sum of the number and 9, we should have

$$\frac{6}{x+9}$$

1.4 Exercises

Evaluate for the given values of the variables.

1. $9x + 4$ for $x = 15$
2. $6x - 3$ for $x = 12$
3. $4x + 2y$ for $x = -2, y = -9$
4. $5x - y$ for $x = -1, y = -6$
5. $3(7x + 4)$ for $x = 3$
6. $5(2x + 5)$ for $x = -2$
7. $x^2 - 6x + 2$ for $x = 2$
8. $2x^2 - 5x + 1$ for $x = -3$
9. $(x + h)^2 - 6(x + h)$ for $x = -4, h = 0.2$
10. $(x + h)^2 + 2(x + h)$ for $x = -3, h = 0.1$
11. $a^2 + b^2 - a^2b^2$ for $a = -3, b = -2$
12. $a^2b^2 + a^3 - b^2$ for $a = -2, b = 4$
13. $\frac{x^2}{y} - \frac{y^2}{x}$ for $x = 3, y = -2$
14. $\frac{2x}{4y} + \frac{x}{xy} + x$ for $x = -1, y = 2$
15. $\frac{2}{3}b + \left(\frac{1}{2}c - a^2\right)$ for $a = -3, b = 6, c = -12$
16. $\frac{1}{8}a - \left(\frac{3}{2}b - c + ab\right)$ for $a = -16, b = -4, c = -2$
17. $(a + c^2) \cdot \left(\frac{b^2+2d}{a}\right)$ for $a = 2, b = 4, c = -5, d = -3$
18. $(c + ab) \cdot \left(\frac{cb-d}{abd}\right)$ for $a = -2, b = 1, c = -2, d = -1$
19. $\left(\frac{c-3a}{b}\right) \div \left(a + \frac{b^2}{cd}\right)$ for $a = 2, b = -3, c = -1, d = -3$
20. $\left(\frac{d^2-ab}{ab}\right) + \left(ac - \frac{ab}{cd}\right)$ for $a = -2, b = -4, c = 3, d = -2$

Simplify.

21. $7a + 5a$

22. $4x - 6x$

23. $-3x + 7x - 8y$

24. $2a - 5b + 3a$

25. $3x^2 + 4y^2 - 2x^2 - 3y^2$

26. $2a^2 + 3b^2 - 4b^2 - 3a^2$

27. $5xy - 7x^2y + 8yx + 4xy^2$

28. $4ba - 3b^2a + 5ab + 7ab^2$

29. $a^2 + b^2 + c^2 + 2ab - 4a^2 - 2ba + 2c^2 - 2b^2$

30. $z^2 + x^2 + yx - x^2 - xy + z^2 - x^2$

31. $2(2 - 3x)$

32. $5(2x - 4)$

33. $-8(5y + 2)$

34. $-4(2y - 7)$

35. $2(4x - 6) + 5x + 8$

36. $4(x - 3) - x + 6$

37. $-2(4a + 5b - 11c)$

38. $6(2x + y - 12)$

39. $5 - (x + 5)$

40. $x - (3x + 4)$

41. $4 - (3x - 6)$

42. $-3 - (x - 3)$

43. $7y - 8(2y - 13)$

44. $5x - (4x + 5y)$

45. $2(3z - 5) + 9(4z + 3)$

46. $6(z - 2) + 3(3z + 2)$

47. $4b - 2(6a + 6b - 14c)$

48. $7z - (6x + 2y - z)$

49. $2(x - 2) - (2x + 4)$

50. $-(2x - 3) - (4x + 5)$

51. $6x - (2x + 3) + 4(x + 5)$

52. $x - (2x + 3) + 3(2x + 1)$

53. $3[2 - 5(x + 1)]$

54. $2[2 - 3(2x + 3)]$

55. $-4(3x + 2) + 4[1 - 3(x + 1)]$

56. $-2(3x - 3) + 3[2 - 2(3x + 1)]$

57. $2[3x - (2x - 1)] - 2$

58. $x - [x - (x - 1)] - 1$

59. $(3xy + yz) - (7y^2z - xz) - (2xy - 4y^2z)$

60. $(xyz + yz) - (y^2z - xzy) - (xy - 2y^2z)$

61. $(8x^2y - 4yz) - (3y^2z - 2xz) - (x^2y + 2y^2z)$

62. $(x^2 - 3yz) - (8y^2z - xz) - (3x^2 + 5y^2z)$

63. $(4a^2b + 2bc) - (5bc^2 - 2a^2c) - (2a^2b - 3bc^2)$

64. $(a^2c + 4bc) - (bc^2 - 2a^2c) - (4a^2b - bc^2)$

65. $3(2x - 1) - (4x + 1) - 2(5x - 6)$

66. $-(4x - 3) - 2(x + 2) - (6x - 1)$

67. $5(x - 2) - 2(4x - 3) - (x + 4)$

68. $3(3x - 2) - 3(x - 2) - (3x + 4)$

69. $6(x - 1) - 2(x - 3) + 3(2x - 7) - 5(2x - 4)$

70. $4(2x - 1) - (2x - 1) + 5(2x - 7) - (12x - 4)$

Translate into a variable expression.

71. a number increased by 8

72. 15 times a number

73. 14 less than a number

74. the quotient of 7 and a number

75. 8 more than 7 times a number

76. 2 less than 5 times a number

77. 5 less than two times a number

78. 6 more than thrice a number

79. the square of a number

80. the cube of a number

81. the sum of a number and the product of the number and 5

82. the product of a number and the sum of 4 and the number

83. the ratio of a number to the sum of the number and 8

84. a number decreased by the quotient of twice the number

85. the difference between the cube of a number and the number

86. 1 less than a number squared times 5

87. the difference between 10 and the quotient of a number and 4

88. the quotient of thrice a number and the number

89. 4 less than thrice the sum of a number and 5

90. 6 more than the sum of the cube of a number and twice the number