

1.2 Rational Numbers

Now that we have a handle on the integers, we will turn our attention to other types of numbers.

We start with the following definitions.

Definition: Rational Number- any number that can be written $\frac{a}{b}$ where a and b are integers.

For example:

$$-\frac{2}{7}, \frac{8}{3}, 5, -2.9$$

So, we can see, all decimals and fractions are Rational Numbers.

Definitions:

Irrational Number- any number which cannot be written as a fraction

Real Numbers- the collection of all rational and irrational numbers

For the time being, we will not work with irrational numbers, other than the number π . We will deal with them much later in the textbook.

For now, we want to concentrate on only the rational numbers, that is to say, the positive and negative fractions and decimals.

We should know that fractions and decimals actually represent the same numbers. If this is truly so, then we need to be able to switch back and forth between them.

It is fairly simple to convert between fractions and decimals. We will take a look at this in the following example.

Example 1:

- a. Write $\frac{15}{16}$ as a decimal. b. Write 3.45 as a fraction.

Solution:

- a. To write a fraction as a decimal, we need to realize that the fraction bar is the same thing as a division symbol. So we can view $\frac{15}{16}$ as actually being $15 \div 16$.

Once we do this, we just long divide as usual. As it turns out, using standard long division we get

$$\begin{array}{r}
 .9375 \\
 16 \overline{) 15.0000} \\
 \underline{144} \\
 60 \\
 \underline{48} \\
 120 \\
 \underline{112} \\
 80 \\
 \underline{80} \\
 0
 \end{array}$$

So, $\frac{15}{16} = 0.9375$.

- b. This time, to write a decimal as a fraction, we need to just drop the decimal point and put the decimal part over the last place value that is implied by the decimal, in this case that is the hundredth place. Then we reduce.

We get

$$3.45 = 3 \frac{45}{100}$$

Remember, to reduce a fraction, we simply divide out common values from the numerator and denominator. In this case, we can take a 5 out of the top and bottom. This gives

$$3 \frac{45}{100} = 3 \frac{9}{20}$$

As it turns out, there is another type of rational number that is also equivalent to fractions and decimals. It is the percent.

Definition: Percent- part of 100, %

So, 5% is 5 parts out of 100 or $\frac{5}{100} = \frac{1}{20}$

Since fractions, decimals, and percents are just three different ways of looking at the same number, we should be able to convert between these three different forms of a number.

In light of the fact that percents are “out of 100”, we use the following set of processes to convert between fractions, decimals and percents.

Converting Between Fractions, Decimals and Percents

- To write a percent as a decimal, move the decimal two places left and drop % symbol.
- To write a percent as a fraction, drop % and put over 100. Then reduce.
- To write a decimal as a percent, move the decimal two places right and add % symbol.
- To write a fraction as a percent, change it into a decimal first, then to a percent.

Let’s try this in the next several examples.

Example 2:

Convert to a decimal.

a. 12%

b. 1.2%

Solution:

- a. To convert from percent to decimal, we simply remove the % symbol and move the decimal twice to the left. This gives us $12\% = 0.12$
- b. Again, we remove the % symbol and move the decimal twice left to get $1.2\% = 0.012$.

Example 3:

Convert to a fraction.

- a. 15%
- b. 10%

Solution:

- a. To convert from percent to fraction, we remove the % symbol, put the number over 100 and reduce. We get

$$\begin{aligned} 15\% &= \frac{15}{100} && \text{Take a 5 out of top and bottom} \\ &= \frac{3}{20} \end{aligned}$$

- b. Again, we remove the % symbol, put over 100 and reduce. We have

$$\begin{aligned} 10\% &= \frac{10}{100} && \text{Take a 10 out of top and bottom} \\ &= \frac{1}{10} \end{aligned}$$

Example 4:

Convert to a percent.

- a. 0.85
- b. 0.034
- c. $\frac{3}{4}$

Solution:

- a. To convert from decimal to percent, we move the decimal twice to the right, then attach a % symbol. We get $0.85 = 85\%$
- b. Again, we move the decimal twice right and attach the % symbol. We get $0.034 = 3.4\%$.
- c. Lastly, to convert from fraction to percent, we convert from fraction to decimal first, then to percent. To convert from fraction to decimal, recall, we divide $3 \div 4$. We get

$$\begin{aligned} \frac{3}{4} &= 0.75 \\ &= 75\% \end{aligned} \quad \begin{array}{l} \text{Move the decimal twice right and} \\ \text{attach \% symbol} \end{array}$$

We will use percents off and on throughout the remainder of this textbook. However, for now, we want to just take a look at fractions and decimals for the remainder of this section.

Now that we have a basic idea of the rational numbers, we need to turn our attention to the basic operations on the rational numbers, just like we did with the integers.

We start with

Adding/Subtracting Fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{and} \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

The idea is, of course, that in order to add or subtract fractions, we must have the same denominator. If we do not, we have to make the fractions have the Least Common Denominator (or LCD) and then add or subtract numerators using the ideas we discussed in the last section.

The LCD is simply the least common multiple of the denominators. That is, the LCD is the smallest number that both denominators divide evenly into.

With this being said, let's take a look at an example.

Example 5:

Perform the indicated operations.

a. $\frac{1}{2} + \frac{3}{8}$

b. $-\frac{2}{3} - \left(-\frac{11}{18}\right)$

c. $-\frac{5}{6} + \frac{3}{4} - \frac{7}{8}$

Solution:

- a. To add two fractions together, we need to have the same denominator. So, we need to start by finding the LCD.

Since the smallest number that both 2 and 8 would divide evenly into is 8, the LCD must be 8.

Now, we make each denominator into an 8 by multiplying the numerator and denominator by whatever it takes to get each denominator to become an 8. In this case, we multiply the top and bottom of the first fraction by 4.

We then add numerators over the common denominator. We get

$$\begin{aligned} & \frac{1}{2} + \frac{3}{8} \\ &= \frac{1 \cdot 4}{2 \cdot 4} + \frac{3}{8} \\ &= \frac{4}{8} + \frac{3}{8} \\ &= \frac{7}{8} \end{aligned}$$

- b. The first thing we should do is a "double switch" like we saw in the last section. Then we will combine the numerators over the LCD which is clearly 18 here.

We get

$$\begin{aligned}
& -\frac{2}{3} - \left(-\frac{11}{18}\right) && \text{Double switch the negatives} \\
& = -\frac{2}{3} + \left(+\frac{11}{18}\right) \\
& = -\frac{2 \cdot 6}{3 \cdot 6} + \frac{11}{18} && \text{Multiply by 6 to get both bottoms to} \\
& && \text{have an 18} \\
& = -\frac{12}{18} + \frac{11}{18} && \text{Add numerators, } -12 + 11 = -1 \\
& = -\frac{1}{18}
\end{aligned}$$

- c. Again, we need to find the LCD. Here the LCD is 24 (the smallest number that 6, 4 and 8 divide evenly into. So, as above, multiply each fraction on top and bottom by whatever it takes to make each denominator into a 24.

We have

$$\begin{aligned}
& -\frac{5}{6} + \frac{3}{4} - \frac{7}{8} \\
& = -\frac{5 \cdot 4}{6 \cdot 4} + \frac{3 \cdot 6}{4 \cdot 6} - \frac{7 \cdot 3}{8 \cdot 3} && \text{Multiply each top and bottom to get} \\
& && \text{a 24 on each denominator} \\
& = -\frac{20}{24} + \frac{18}{24} - \frac{21}{24} && \text{Add numerators, } -20 + 18 - 21 = -23 \\
& = -\frac{23}{24}
\end{aligned}$$

We also need to be able to add and subtract decimals.

Adding/Subtracting Decimals

Just line up the decimal point and use the rules for adding and subtracting as before.

This is very easy to do, but we need to be sure we are carefully following the rules we established in the last section.

Example 6:

Perform the indicated operations.

a. $-32.1 - 6.7$

b. $2.09 - 6.72 - 5.4$

Solution:

- a. The first thing we need to do is decide if we need to add or subtract the numbers. In this case, since they both have the same sign (by attaching the - to each number after it, as we did in the last section) we need to add the numbers together (they work together).

So, we line up the decimal points, and add like we did before.

$$\begin{array}{r} 32.1 \\ +6.7 \\ \hline 38.8 \end{array}$$

Since both numbers were negative, the answer must be negative (again because they work together, like we saw in the last section).

So our answer is -38.8

- b. Here, we do these numbers two at a time. Let's start with the first two, $2.09 - 6.72$. Since they have different signs, we have to subtract the numbers and add the sign of the "larger" number, a negative in this case.

So we get

$$\begin{array}{r} 6.72 \\ -2.09 \\ \hline 4.63 \end{array}$$

Now our problem is $-4.63 - 5.4$. Since the signs are the same here, we add the numbers and attach the common sign.

We get

$$\begin{array}{r} 4.63 \\ +5.4 \\ \hline 10.03 \end{array}$$

So the answer is -10.03.

Lastly, we need to take a look at multiplying and dividing rational numbers.

Multiplying/Dividing Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad \text{and} \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

The concept here is to multiply, we multiply straight across. However, we generally like to try and reduce (or cancel) before we multiply across to make the problem easier.

For division, we "flip" (or more technically "invert") the fraction after the division symbol and turn the problem into multiplication, then proceed as usual.

Let's take a look at some examples.

Example 7:

Perform the indicated operations.

a. $-\frac{2}{9} \cdot \frac{3}{14}$

b. $\left(-\frac{5}{12}\right) \div \left(-\frac{15}{32}\right)$

Solution:

- a. To multiply fractions, we start by trying to reduce any numerator with any denominator. Then we multiply straight across, keeping in mind the rules for multiplication. In this case, we have a negative times a positive. Therefore our answer is negative.

We have

$$\begin{array}{r} 1 \ 1 \\ \frac{2}{9} \cdot \frac{3}{14} \\ \hline \frac{3}{7} \\ = -\frac{1}{21} \end{array}$$

- b. When dividing, we start with “flipping” the second fraction, then multiply as we just saw in part a. Keep in mind, we will end up with a positive in the end, since a negative divided by a negative is a positive.

This gives

$$\begin{aligned} & \left(-\frac{5}{12}\right) \div \left(-\frac{15}{32}\right) \\ &= \left(-\frac{5}{12}\right) \cdot \left(-\frac{32}{15}\right) \\ &= \frac{8}{9} \end{aligned}$$

Now for decimals we just need to properly deal with the decimal point and multiply or divide as usual.

Multiplying Decimals

Multiply as we did with integers and give the product the total of all decimal places.

This means, multiply as usual, then give the answer the number of decimal places of all of the numbers you multiplied, added together.

Dividing Decimals

Move the decimal in the divisor, then move the decimal the same in the dividend. Pull it straight up and divide as usual.

Let's look at an example.

Example 8:

Perform the indicated operations.

a. $(-0.8)(4.2)$

b. $0.0976 \div 0.042$, round to the hundredth

Solution:

- a. To multiply, we start with standard multiplication of the numbers, that is 42 times 8, which is 336.

Now, the answer has the total of all the decimal places in the problems, meaning since the numbers each had one decimal place, the answer must have 2 decimal places (1 place + 1 place = 2 places). Then we attach the proper sign, a negative in this case.

So we get an answer of -3.36.

- b. Lastly, we start by setting up the long division and move the decimals as outlined above.

We get

$$\begin{array}{r} 0.042 \overline{)0.0976} \\ 42 \overline{)97.6} \end{array}$$

Now long divide, adding enough zeros until we can properly round to the thousandth, that means we need to go out to the ten thousandths since we have to look at the next place value below to determine how to round.

This gives

$$\begin{array}{r} 2.3238 \\ 42 \overline{)97.6000} \\ \underline{84} \\ 136 \\ \underline{126} \\ 100 \\ \underline{84} \\ 160 \\ \underline{126} \\ 340 \\ \underline{336} \\ 4 \end{array}$$

So, rounding gives us 2.324.

1.2 Exercises

Write as a decimal.

1. $\frac{4}{5}$

2. $\frac{1}{2}$

3. $\frac{3}{4}$

4. $\frac{1}{4}$

5. $\frac{7}{8}$

6. $\frac{5}{8}$

7. $1\frac{11}{16}$

8. $2\frac{13}{32}$

9. $2\frac{17}{20}$

10. $1\frac{19}{50}$

Write as a reduced fraction.

11. 0.8

12. 0.5

13. 2.4

14. 6.8

15. 3.12

16. 7.15

17. 8.65

18. 5.34

19. 10.87

20. 12.13

Convert to decimal.

21. 12%

22. 15%

23. 1.5%

24. 6.2%

25. 125%

26. 240%

27. 0.32%

28. 0.11%

Convert to fraction.

29. 20%

30. 15%

31. 35%

32. 16%

33. 2%

34. 5%

35. 124%

36. 230%

Convert to percent.

37. $\frac{1}{2}$

38. $\frac{4}{5}$

39. $\frac{7}{8}$

40. $\frac{1}{20}$

41. $\frac{9}{16}$

42. $\frac{9}{32}$

43. 0.18

44. 0.24

45. 0.064

46. 0.073

47. 12

48. 13

Perform the indicated operations.

49. $\frac{5}{8} - \frac{5}{6}$

50. $\frac{1}{9} - \frac{5}{27}$

51. $-\frac{7}{12} - \left(-\frac{7}{8}\right)$

52. $-\frac{5}{8} - \left(-\frac{11}{12}\right)$

53. $-\frac{3}{4} - \frac{5}{8}$

54. $\frac{5}{12} - \frac{11}{15}$

55. $\frac{1}{2} - \frac{3}{8} - \left(-\frac{1}{4}\right)$

56. $-\frac{5}{16} + \frac{3}{4} - \frac{7}{8}$

57. $-\frac{3}{8} - \frac{5}{12} - \frac{3}{16}$

58. $\frac{1}{2} - \frac{3}{8} - \left(-\frac{1}{4}\right)$

59. $\frac{1}{2} - \frac{3}{8} + \frac{5}{12}$

60. $\frac{1}{3} - \frac{1}{5} - \frac{1}{7}$

61. $\frac{1}{2} \cdot \left(-\frac{3}{4}\right)$

62. $\frac{4}{5} \cdot \left(-\frac{13}{25}\right)$

63. $-\frac{2}{9} \cdot \left(-\frac{3}{14}\right)$

64. $-\frac{1}{12} \cdot \left(-\frac{3}{4}\right)$

65. $-\frac{3}{8} \div \frac{7}{8}$

66. $\frac{1}{3} \div \left(-\frac{1}{2}\right)$

67. $\left(-\frac{3}{4}\right) \div \left(-\frac{7}{40}\right)$

68. $-\frac{7}{18} \div \frac{14}{9}$

69. $-\frac{2}{3} \div 4$

70. $\left(-\frac{3}{8}\right) \div \left(-\frac{5}{16}\right)$

71. $2.3 - 6.8$

72. $-8.9 - 6.78$

73. $9.32 - (-3.6)$

74. $-3.2 - (-7.89)$

75. $14 + (-3.4) - (-5.67)$

76. $15.4 - 18 + (-23.775)$

77. $3.9 \cdot (-9.4)$

78. $-5.4 \cdot (7.008)$

79. $(-42.1)(-0.0034)$

80. $3.44(-2.56)$

81. $-0.01(5.62)(-0.0002)$

82. $0.23(-8.54)(-0.0022)$

83. $-2.34 \div (-8.3)$ round to the thousandth

84. $-24.18 \div 0.0034$ round to hundredth

85. $0.00245 \div (-1.485)$ round to ten thousandth

86. $(-0.00365) \div (-0.336)$ round to ten thousandth