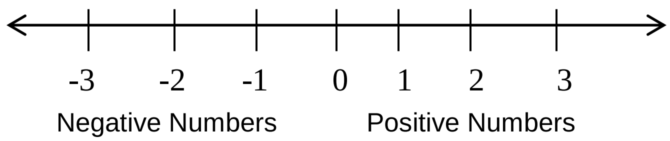
1.1 Integers

To begin this textbook, we need to start with a refresher of the topics of numbers and numbering systems.

We will start, here, with a recap of the simplest of numbering systems, the integers. We, then, build from there in the next sections and following.

Definition: **Integers**- $\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$
We can show them on a number line:



The number line is useful for a variety of topic we will be looking at. We can use the number line for representing numbers geometrically, for one, but we can also use the number line to establish the order of numbers.

Let's consider the following definition.

Definition: **Inequality Symbols**-
-If a and b are two numbers and a is to the left of b on the number line, then a is less than b, written a<b
-If a and b are two numbers and a is to the right of b on the number line, then a is greater than b, written a>b

We will use this definition in the following example.

Example 1:

Place the correct symbol, < or >.

a. -14___16 b. -17___-15 c. 83___-200

Solution:

a. First, looking at the number line, we can see that -14 is to the left of 16. So by the definition above, -14 is less than 16. So we get

-14<16

b. This time, -17 is to the left of -15 on the number line. So, -17 is less than -15. This gives

-17<-15

c. Lastly, on the number line 83 would be far to the right of -200. Therefore, 83 is greater than -200. So

83>-200

Another foundational topic which uses a geometric idea is that of the distance a number is, on the number line, away from zero.

We have the following definition.

<u>Definition</u>: **<u>Absolute Value</u>**- the distance a number is from zero, written $|a|$

Using this definition, we can evaluate some expressions which contain an absolute value.

<u>Example 2</u>:

Evaluate.

a. $|-4|$ b. $|8|$ c. $-|-2|$ d. $-|16|$

Solution:

a. Since the absolute value represents the distance the number is from zero, $|-4|$ must represent how far -4 is away from zero.

   Clearly, -4 is 4 spaces from zero. Therefore, $|-4| = 4$.

b. Again, since the absolute value is the distance from zero, and 8 is clearly 8 spaces from zero, $|8| = 8$.

c. This time, the problem is a little more difficult. We need to start with $|-2|$. The negative on the outside we will deal with later.

   So, since -2 is 2 spaces from zero, $|-2| = 2$. However, we have an extra negative on the outside of the absolute value. Therefore,

$$-|-2|$$
$$= -2$$

d. Just like in part c above, we start with the $|16| = 16$. Then the extra negative on the outside gives us $-|16| = -16$.

Now that we have the basics of the integers, we need to look at the basic operations on the integers. By basic operations we mean, add, subtract, multiply and divide.

We start with adding.

<u>Adding Integers</u>

The idea behind adding is a simple one. We will add or subtract the numbers and then attach the appropriate sign. The way it works is,

- If two numbers have the same sign, they work together. This means you add the numbers then attach the common sign.

- If two numbers have different signs, they work against each other. This means, you subtract the numbers and attach the sign of the number with the larger absolute value.

Let’s see an example to help clarify these rules.

Example 3:

Add.

a. -6 + (-9) b. -9 + 2 c. 19 + (-91) d. 201 + (-56) + (-123) + 13

Solution:

a. Since the 6 and the 9 have the same sign, they will work together. This means we add them to get 6 + 9 = 15, then we attach the common sign. Because they are both negative, we get -6 + (-9) = -15.

b. This time the numbers have different signs. This means (even though it is an addition problem) we have to subtract them. Since they work against each other.
We get 9 - 2 = 7.

   Now, since the 9 has a larger absolute value, and it is negative, in a sense it takes over the problem. So, this means our answer must be negative.

   Therefore, we have -9 + 2 = -7.

c. As in part b above, the numbers have different signs. So we start by subtracting them, 91 - 19 = 72. Since the 91 has a larger absolute value, and he is negative, the answer must be -72.

d. In this problem, we have more than just two numbers to work with. The concept is the same, however. We just need to do the problem two numbers at a time as follows.

$$\underbrace{201 + (-56)} + \underbrace{(-123) + 13}$$
$$= \underbrace{145 \quad + \quad (-110)}$$
$$= 35$$

For subtraction, there are numerous different techniques that can be used. We will simply present only one, for the sake of simplicity.

Subtracting Integers

To subtract, we simply view every problem as an addition problem. What this means is, whatever sign in in front of a number, the sign is attached to the number. Then, we simply add as we learned above.

The exception to that is if we are subtracting a negative, we switch both signs to positive and again add like before. These situations we call a “double-switch”.

Let’s see how this works in the following example.

Example 4:

Subtract.

a. 6 – 9 b. -4 – (-2) c. 7 – 8 – (-1) d. 17 – (-17) – 14 – 21

Solution:

a. As stated, we want to actually view the problem as an addition problem. To do so, we want to visualize the 6 as a +6 and the subtract 9 as a -9. This way, if we attach the sign before the number to the number itself, we can turn every problem into an addition problem, as well as move numbers around within a problem. We just need to make sure we keep the sign with the number.

   So, once we view it as addition we have 6 - 9 = 6 + (-9) = -3

b. In this case, we have two negatives right next to each other. So, here, we do the "double switch", meaning, switch both signs to positives. This gives us -4 - (-2) = -4 + (+2).

   Now adding like before we get -4 + 2 = -2

c. The first thing we need to do is go through the problem and take care of any "double switches". This gives us

   7 – 8 – (-1)

   =7 – 8 + (+1)

   Now, view the negative as being attached to the 8 and add like we did in example 3.

   7 – 8 + (+1)

   =-1 + 1

   = 0

d. Finally, we start with "double switches" and attach signs followed by adding. We get

   17 – (-17) – 14 – 21

   =17 + (+17) – 14 – 21

   = 34 - 35

   = -1

Now, let's mix adding and subtracting together in the following.

Example 5:

Perform the indicated operations.

a. 25 – 18 + (-39) – (-54) + 15 – 80

b. -18 – 49 + 84 – 27 – (-31)

Solution:

a. Since subtraction is just a specialized version of addition, we can treat all of this the same. We begin with our "double switches" then attach signs and add in pairs until we are down to a single number. We get

   25 – 18 + (-39) – (-54) + 15 – 80

   =25 – 18 + (-39) + (+54) + 15 – 80

   = 7 + 15 - 65

$= \quad 22 \quad -65$

$= \quad -43$

b. Again, we start with "double switches" adding as usual.

-18 – 49 + 84 – 27 – (-31)

=-18 – 49 + 84 – 27 + (+31)

= -67 + 57 + 31

= -10 + 31

= 21

Like with subtraction above, there are several different methods that we could learn to do multiplying and dividing of integers. We will simply look at one.

Multiplying and Dividing Integers

To multiply and divide numbers, what we need to know is that we simply multiply or divide the values as we have learned in the past, and then we use the following rules for the signs.

We will use a fraction bar for the division symbol since, as we will see, they are the same thing.

$$+\cdot+=+ \qquad \frac{+}{+}=+$$
$$+\cdot-=- \qquad \frac{+}{-}=-$$
$$-\cdot+=- \qquad \frac{-}{+}=-$$
$$-\cdot-=+ \qquad \frac{-}{-}=+$$

The idea is, whether multiplying or dividing, an even number of negatives makes a positive and an odd number of negatives makes a negative.

Let's take a look at some examples.

<u>Example 6</u>:

Multiply.

a. (-8)3 b. (-4)(-3) c. (-14)9(-1) d. (-3)7(-2)(-8)

Solution:

a. To multiply we simply start by multiplying the numbers 8 x 3 = 24. Then we attach the sign according to the chart given above. Since a negative times a positive is a negative,

(-8)3 = -24

b. Again, we multiply 4 x 3 = 12. Negative time negative is a positive, so we have

(-4)(-3) = 12

c. Here, we start by multiplying all of the numbers together. 14 x 9 x 1 = 126.

Since we have an even number of negatives, the answer must be positive. Therefore,

$$(-14)9(-1) = 126$$

d. Lastly, we, as above, multiply all of the numbers together 3 x 7 x 2 x 8 = 336. Since we have an odd number of negatives, the answer is negative. We have

$$(-3)7(-2)(-8) = -336$$

<u>Example 7</u>:

Divide.

a. $\frac{-36}{-9}$ b. $\frac{-114}{6}$ c. $\frac{128}{-4}$

Solution:

a. Similar to how we did multiplication above, we start with dividing the numbers, then we attach the proper sign. So, 36 divided by 9 is 4, and negative divided by negative is positive. So

$$\frac{-36}{-9} = 4$$

b. Again, we divide 114 by 6 to get 19. Negative divided by positive is negative. So we have

$$\frac{-114}{6} = -19$$

c. Finally, we have 128 divided by 4 is 32. Positive divided by negative gives

$$\frac{128}{-4} = -32$$

Lastly, we have a few properties for division (and consequently fractions) that we will need as we progress through the textbook.

We state them here for future use.

<u>Miscellaneous Properties</u>

$-\frac{a}{b} = \frac{a}{-b} = \frac{-a}{b}$ $\frac{0}{a} = 0$ $\frac{a}{0} = undefined$ $\frac{a}{a} = 1$ $\frac{a}{1} = a$

1.1 Exercises

Place the correct symbol, < or >.

1. -2___8
2. -5___-9
3. -12___-36
4. 18___-4
5. -46___0
6. 0___-16
7. -56___-120
8. 83___-72
9. 267___-267
10. 128___-128
11. 532___-621
12. -754___732

Evaluate.

13. $|-2|$
14. $|-7|$
15. $|10|$
16. $|34|$
17. $|0|$
18. $|-13|$
19. $|-56|$
20. $|23|$
21. $-|-54|$
22. $-|30|$
23. $-|21|$
24. $-|-1|$
25. $-|171|$
26. $-|-452|$
27. $|-1{,}030|$
28. $-|987|$

Perform the indicated operations.

29. $-7+12$
30. $-15+(-2)$
31. $5+(-18)$
32. $12+(-19)$
33. $-19-12$
34. $45-67$
35. $97-132$
36. $-123-21$
37. $-12-(-57)$
38. $52-(-38)$
39. $73-(-27)$
40. $-18-(-34)$
41. $-4+43-24$
42. $21-18+(-23)$
43. $-56+48-65-21$
44. $-65+87-69-45$
45. $54+(-65)-24-128+154$
46. $25+(-96)+101-(-121)+98$
47. $100-(-12)-(-67)+(-200)$
48. $67+(-145)-24-714$
49. $657+(-345)-234-974$
50. $134-(-52)-(-72)+(-120)$
51. $-1000-(-243)+(-184)+671-782$
52. $-980+(-213)-(-432)+721-(-82)$
53. $-4\cdot 5$
54. $-6\cdot -7$
55. $(-8)(-8)$
56. $(7)(-5)$
57. $10(-1)$
58. $(-15)(-2)$
59. $-1(-2)(-4)$
60. $2(-3)(-6)$
61. $4(-2)(6)(-5)$
62. $-2(-2)(3)(-3)$
63. $-1(-1)(-1)(-1)$
64. $-3(-3)(-3)(-3)$
65. $\frac{-81}{3}$
66. $\frac{-64}{-4}$
67. $\frac{-156}{-12}$
68. $\frac{-154}{-11}$
69. $\frac{150}{-6}$
70. $\frac{234}{-13}$
71. $\frac{-216}{-9}$
72. $\frac{-136}{17}$
73. $\frac{-35}{-35}$
74. $\frac{-27}{-27}$
75. $\frac{-45}{0}$
76. $\frac{-78}{0}$
77. $\frac{0}{-123}$
78. $\frac{0}{-345}$
79. $\frac{-542}{1}$
80. $\frac{-652}{1}$