13.6 Systems of Inequalities in Two Variables

Now that we can graph all two variable inequalities, we want to turn our attention to graphing systems of inequalities that contain these two variable inequalities.

<u>Definition</u>: **<u>System of Inequalities in Two Variables</u>**- Two or more inequalities involving the same variables.

First, recall graphing a system of linear inequalities.

For a system of linear inequalities, the solution will simply be the region on the rectangular coordinate system where the shading of the inequalities overlap.

<u>Example 1</u>:

Graph the system of inequalities.

a. $\begin{matrix} x - 3y < -3 \\ 3x + y \leq -2 \end{matrix}$　　b. $\begin{matrix} 2x + y > 3 \\ y \leq -2x - 6 \end{matrix}$　　c. $\begin{matrix} x - y \leq 4 \\ 2x - 2y \geq 8 \end{matrix}$

Solution:

a. First, we need to graph both inequalities as on the same plane. It can be easily shown that we get the following

$x - 3y < -3\text{:} \quad \mathrm{m} = \frac{1}{3}, \mathrm{x-int}\text{: } (-3,0), \mathrm{y-int}\text{: } (0,1)$

$3x + y \leq -2\text{:} \quad \mathrm{m} = -3, \mathrm{x-int}\text{: } \left(-\frac{2}{3}, 0\right), \mathrm{y-int}\text{: } (0,-2)$

Using (0,0) for our test point we get

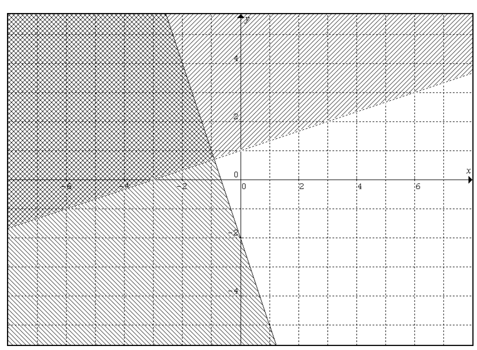
$$\begin{aligned} x - 3y &< -3 \\ 0 - 3(0) &< -3 \\ 0 &< -3 \end{aligned}$$

False

$$\begin{aligned} 3x + y &\leq -2 \\ 3(0) + 0 &\leq -2 \\ 0 &\leq -2 \end{aligned}$$

False

So graphing together we get



The solution to the system is only the region that is "double shaded."

b. So we start, again, by finding all of our usual information to find the graphs.

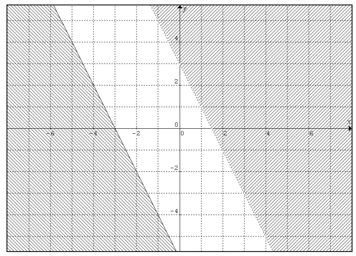
$2x + y > 3\text{:} \quad \mathrm{m} = -2, \mathrm{x-int}\text{: } (3/2, 0), \mathrm{y-int}\text{: } (0,3)$

$y \leq -2x - 6\text{:} \quad \mathrm{m} = -2, \mathrm{x-int}\text{: } (-3,0), \ \mathrm{y-int}\text{: } (0,-6)$

Again, testing (0,0) gives

$2x + y > 3$ | $y \le -2x - 6$
$2(0) + 0 > 3$ | $0 \le -2(0) - 6$
$0 > 3$ | $0 \le -6$
False | False

So we get the graph



Since clearly the shaded regions do not overlap, this system has no solution.

c. Lastly, we find the information we need for the graphs.

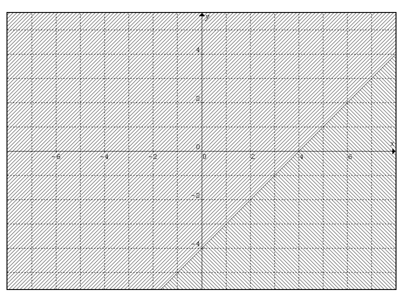
$x - y \le 4$: $\mathrm{m} = 1, \mathrm{x-int}: (4,0)$, $\mathrm{y-int}: (0,-4)$
$2x - 2y \ge 8$: $\mathrm{m} = 1, \mathrm{x-int}: (4,0), \mathrm{y-int}: (0,-4)$

Since we have the same slope and same intercepts, the lines must be the same line.

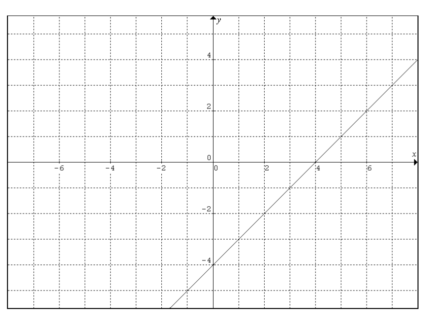
Testing (0, 0)

$x - y \le 4$ | $2x - 2y \ge 8$
0 - 0 ≤ 4 | 2(0)-2(0)≥8
$0 \le 4$ | $0 \ge 8$
True | False

Now let's graph and see what we can deduce.



Since the "shading" part does not overlap, but the lines are both solid, the solution to the system is only the line. So the solution is just the line



As was the case with graphing a single inequality of two variables is done the same as graphing the linear types, with a system, it works likewise. We just need to graph each inequality of the system and see where the shading overlaps.

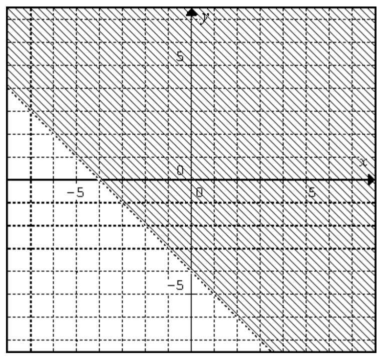
Example 2:

Graph the system of inequalities.

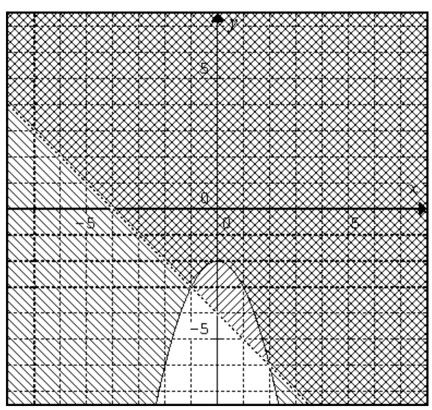
a. $x + y > -4$, $x^2 + y \geq -2$ b. $x^2 + y^2 < 4$, $x^2 - y^2 \geq 1$ c. $y \leq \sqrt{x+4}$, $4(x+5)^2 + y^2 \leq 4$

Solution:

a. We begin by graphing each inequality as we have learned before.
The graph of $x + y > -4$ is a line with slope -1, x-intercept of (-4,0) and y-intercept of (0,-4). Testing (0,0) gives 0 + 0 > -4 which is true. So we get

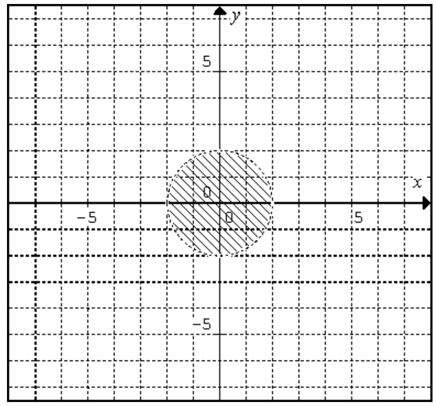


For $x^2 + y \geq -2$ we have a parabola ($y \geq -x^2 - 2$) with vertex of (0,-2), no x-intercepts and a y-intercept of (0,-2). Testing (0,0) gives $0^2 + 0 \geq -2$, which is true. Adding that to the graph of the first part we get

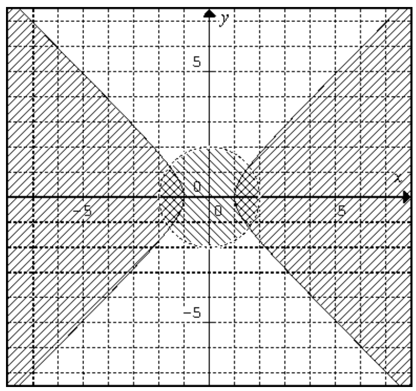


The solution, therefore, is the region where the shading is overlapping.

b. Again, we start with graphing each inequality. $x^2 + y^2 < 4$ is a circle of radius 2 centered at the origin. Testing (0,0) gives 0 < 4 which is true. So we get

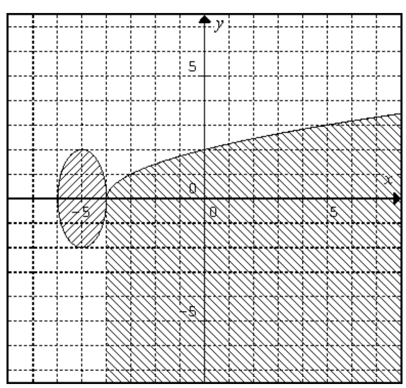


Now, graphing $x^2 - y^2 \geq 1$ is just a hyperbola, also centered at the origin with a horizontal axis. Testing (0,0) we get $0 \geq 1$ which is false. So we have



Again, the solution is the region where the shading overlaps.

c. Lastly, as we did with the others, we graph each inequality on the same axis. $y \leq \sqrt{x+4}$ is a square root function which is shifted 4 units left and $4(x+5)^2 + y^2 \leq 4$ is an ellipse centered at (-5,0). Testing each at (0,0) to determine the shading is simple and is left to the reader.
We get the following



Oddly enough, we see that the two graphs only overlap at a single point, (-4,0). Therefore, the solution is just the point (-4,0).

Keep in mind, as with all systems, there is a wide range of potential solutions. Including situations where there is no solution (the graphs do not overlap at all), one single point (as we saw above, or even situations where the overlap is just one of the regions from only one of the graphs.

Because of the wide variety of possibilities, its best to just take each system as it comes and let the shading tell you what the solution is.

13.6 Exercises

Graph the system of inequalities.

1. $y > 6x - 11$, $2x + 3y < 7$
2. $7x + 2y > -13$, $x - 2y \geq 11$
3. $2x - 3y < -1$, $y \leq x - 1$
4. $x + 2y \leq -4$, $4y \geq 3x + 12$
5. $y \geq -3x + 5$, $5x - 4y \leq -3$
6. $x + y < 0$, $3x + y > -4$
7. $3x + 3y < -3$, $y > -5x - 17$
8. $x - 3y \leq 6$, $2x - 6y \geq 6$
9. $6x - 4y > 2$, $3x - 2y < 4$
10. $x - 4y < -3$, $3x + 2y \leq -2$
11. $x^2 + y^2 \leq 9$, $y - x < 2$
12. $x^2 + 9y^2 \leq 9$, $y - 2x < -1$
13. $x^2 + y^2 < 25$, $y \leq -2$
14. $x^2 + y^2 \leq 4$, $2y - x < 5$
15. $25x^2 + 9y^2 \geq 225$, $5x + 3y < 15$
16. $4x^2 + y^2 \leq 4$, $y^2 - x^2 > 1$
17. $x^2 + y^2 < 25$, $x^2 - y^2 > 4$
18. $x^2 - y^2 \leq 25$, $y^2 \leq -2x + 5$
19. $x^2 + y^2 \geq 36$, $y \leq -x^2 + 1$
20. $x^2 + 4y^2 \leq 16$, $y - x^2 \geq 5$
21. $x^2 + y^2 \leq 25$, $y^2 \leq x + 5$
22. $x^2 - y^2 \leq 25$, $x^2 + 3 > y$
23. $y \leq \frac{2}{x}$, $x + y < 1$
24. $y \leq \frac{1}{x} - 1$, $2x + y \geq 1$
25. $y < -|x| + 2$, $y > |x| - 2$
26. $y > |x + 1| + 2$, $y \leq -|x - 1|$
27. $y > -(x + 1)^3$, $x > (y - 2)^2$
28. $y < -\sqrt{x + 2}$, $y > (x - 2)^2$
29. $y < 2^x$, $y \geq 4^{-x} - 2$
30. $y > -4^x + 1$, $y \geq 3^{x-2} - 2$
31. $y \leq log_2(x + 1)$, $y > 5 - log_2(x - 3)$
32. $y \leq -log_3(x - 2)$, $y > log_4(-x)$
33. $(x - 1)^2 + 4y^2 \leq 4$, $(x - 3)^2 + y^2 \leq 4$
34. $(x - 1)^2 + (y + 1)^2 > 4$, $(x - 1)^2 + 4(y + 1)^2 \leq 16$
35. $y < -\sqrt{x}$, $(x - 3)^2 + y^2 > 4$
36. $y < \frac{1}{x+2} + 1$, $x^2 - y^2 > 4$
37. $\frac{x^2}{4} - \frac{(y-1)^2}{9} < 1$, $\frac{x^2}{9} + \frac{(y-1)^2}{4} > 1$
38. $\frac{(x-1)^2}{9} - \frac{(y-1)^2}{16} < 1$, $(x - 3)^2 - \frac{(y-4)^2}{4} > 1$
39. $y \leq -log_2(x - 1)$, $y^2 \leq x + 5$
40. $y < log_2(x + 4) + 3$, $y > 3^x - 2$
41. $y \leq [\![x]\!]$, $y > [\![x]\!] - 1$
42. $y \leq [\![x + 2]\!]$, $y > [\![x - 1]\!]$