13.2 Solving Larger Systems by Gaussian Elimination

Now we want to learn how to solve systems of equations that have more that two variables. We start with the following definition.

Definition: Row echelon form
A system is in row echelon form when it has a “stair-step” pattern with leading coefficients of 1.

Here is an example of a system that is in row echelon form.

$$\begin{aligned} x+\tfrac{4}{5}y-\tfrac{1}{5}z&=0 \\ y-\tfrac{3}{10}z&=\tfrac{11}{10} \\ z&=3 \end{aligned}$$

The reason we need row echelon form, is that when a system is in row echelon form it is very easy to solve. We simply use a technique called back substituting. To do this we simply take the variables and substitute the value one at a time into the equation above it. We illustrate this technique in the following example.

Example 1:

Solve the system.

$$\begin{aligned} x+\tfrac{4}{5}y-\tfrac{1}{5}z&=0 \\ y-\tfrac{3}{10}z&=\tfrac{11}{10} \\ z&=3 \end{aligned}$$

Solution:
So we need to back substitute to solve the system. Since we can see from the last equation that z = 3, we can substitute this value into the middle equation and solve for y. We get

$$\begin{aligned} y-\tfrac{3}{10}(3)&=\tfrac{11}{10} \\ y-\tfrac{9}{10}&=\tfrac{11}{10} \\ y&=\tfrac{20}{10} \\ y&=2 \end{aligned}$$

Now that we have both y and z, we can substitute those values into the top equation and solve for x. We get

$$\begin{aligned} x+\tfrac{4}{5}(2)-\tfrac{1}{5}(3)&=0 \\ x+\tfrac{8}{5}-\tfrac{3}{5}&=0 \\ x+1&=0 \\ x&=-1 \end{aligned}$$

So now we have x, y and z. We put the solution as an ordered triple of the form $(x,\ y,\ z)$. So the solution to our system is $(-1,\ 2,\ 3)$.

So we can see that having a system in row echelon form can be a very useful thing in seeking the solution to a system of equations.

With that in mind we make the following definition.

Definition: Equivalent systems
Two systems are called <u>equivalent</u> if they have the same solution set.

So if we are able to put a system into row echelon form in a way that keeps a system equivalent, then we could easily solve the system by back substitution. Fortunately for us, we do have such a technique. We use something we call the elementary row operations.

Elementary Row Operations
The following operations on a system of linear equations produces an equivalent system: 1. Interchange two equations 2. Multiply one equation by a non-zero constant 3. Add a multiple of one equation to another to replace the latter equation

Using the row operations to solve a system is called solving by Gaussian Elimination.

<u>Example 2</u>:

Solve by Gaussian Elimination.

a. $$\begin{aligned} x - y + 2z &= -4 \\ 3x + y - 4z &= -6 \\ 2x + 3y - 4z &= 4 \end{aligned}$$

b. $$\begin{aligned} 2x + 6y - 4z &= 8 \\ 3x + 10y - 7z &= 12 \\ -2x - 6y + 5z &= -3 \end{aligned}$$

Solution:

a. To solve a system by Gaussian elimination we need to use our row operations to put the equation into row echelon form. Then we can easily back substitute and solve. Remember, row echelon form has a stair step pattern with leading coefficients of one. With this in mind, we can see that we should leave the first equation on top, since it already has leading coefficient of 1. So, we need to find a way to make the other two equations not have an x term. We use row operation 3. That is, we can multiply the top equation by -3 and add it to the second equation to get a new second equation. To simplify the matter we call the top equation E_1, the middle equation E_2 and the bottom equation E_3. So, we are going to do $-3E_1 + E_2$. This gives

$$\begin{aligned} -3(x - y + 2z) &= -3(-4) \\ 3x + y - 4z &= -6 \end{aligned} \Rightarrow \begin{array}{r} -3x + 3y - 6z = 12 \\ 3x + y - 4z = -6 \\ \hline 4y - 10z = 6 \end{array}$$

So, $4y - 10z = 6$ is our new E_2. So our system now looks like

$$\begin{aligned} x - y + 2z &= -4 \\ 4y - 10z &= 6 \\ 2x + 3y - 4z &= 4 \end{aligned}$$

Now we do the same thing to eliminate the x from E_3. We do $-2E_1 + E_3$. This will become our new E_3. We get

$$\begin{aligned} -2(x - y + 2z) &= -2(-4) \\ 2x + 3y - 4z &= 4 \end{aligned} \Rightarrow \begin{array}{r} -2x + 2y - 4z = 8 \\ 2x + 3y - 4z = 4 \\ \hline 5y - 8z = 12 \end{array}$$

So our system is now

$$
\begin{aligned}
x - y + 2z &= -4 \\
4y - 10z &= 6 \\
5y - 8z &= 12
\end{aligned}
$$

Now, remember, to get row echelon form, we need the leading coefficients to be 1. So we can multiply equation 2 by ¼ to make the leading coefficient 1. So we write ¼E_2 and we get

$$\tfrac{1}{4}(4y - 10z) = \tfrac{1}{4}(12) \Rightarrow y - \tfrac{5}{2}z = \tfrac{3}{2}$$

So our system is now

$$
\begin{aligned}
x - y + 2z &= -4 \\
y - \tfrac{5}{2}z &= \tfrac{3}{2} \\
5y - 8z &= 12
\end{aligned}
$$

We almost have row echelon form. We need only get rid of the 5y in E_3. Just like we did for x, we can use the third row operation. This time we will use E_2. So we use $-5E_2 + E_3$. We get

$$
\begin{array}{c}
-5\left(y - \tfrac{5}{2}z\right) = -5\left(\tfrac{3}{2}\right) \\
5y - 8z = 12
\end{array}
\Rightarrow
\begin{array}{r}
-5y + \tfrac{25}{2}z = -\tfrac{15}{2} \\
\underline{5y - 8z = 12} \\
\tfrac{9}{2}z = \tfrac{9}{2}
\end{array}
$$

This now becomes our new E_3. So our system is

$$
\begin{aligned}
x - y + 2z &= -4 \\
y - \tfrac{5}{2}z &= \tfrac{3}{2} \\
\tfrac{9}{2}z &= \tfrac{9}{2}
\end{aligned}
$$

To finish getting it into row echelon form we simply go $\tfrac{9}{2}$ E_3. We get

$$
\begin{aligned}
x - y + 2z &= -4 \\
y - \tfrac{5}{2}z &= \tfrac{3}{2} \\
z &= 1
\end{aligned}
$$

Now we simply need to back substitute to solve. We proceed as follows.

$$
\begin{aligned}
y - \tfrac{5}{2}(1) &= \tfrac{3}{2} \\
y &= \tfrac{8}{2} \\
y &= 4
\end{aligned}
\qquad\qquad
\begin{aligned}
x - (4) + 2(1) &= -4 \\
x - 2 &= -4 \\
x &= -2
\end{aligned}
$$

So the solution is $(-2,\ 4,\ 1)$.

b. Again, we need to start by getting the system into row echelon form. We can see clearly that if we do $E_1 + E_2$, then we will eliminate one x value relatively easily. So we get

$$
\begin{array}{r}
2x + 6y - 4z = 8 \\
\underline{-2x - 6y + 5z = -3} \\
-z = 5
\end{array}
$$

In fact, this operation eliminated several steps. So this will become our new E_3. We will also, go ahead and insert $-E_3$ just to simplify the matter. Our system is now

$$
\begin{aligned}
2x + 6y - 4z &= 8 \\
3x + 10y - 7z &= 12 \\
z &= -5
\end{aligned}
$$

Now we can go ½E_1 to get the coefficient to be 1. We get

$$\begin{aligned} x+3y-2z&=4\\ 3x+10y-7z&=12\\ z&=-5 \end{aligned}$$

Clearly now we will do -3E_1 + E_2 to replace E_2. We get

$$\begin{aligned} -3(x+3y-2z)&=-3(4)\\ -3x+10y-7z&=12 \end{aligned} \Rightarrow \begin{aligned} -3x-9y+6z&=-12\\ \underline{3x+10y-7z}&\underline{=12}\\ y-z&=0 \end{aligned}$$

So our system becomes

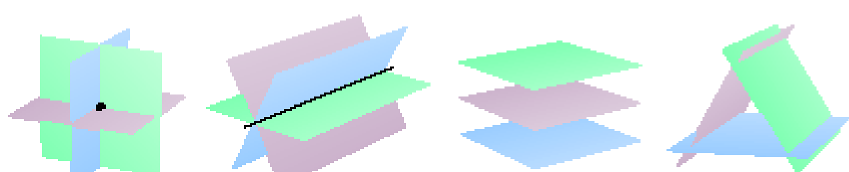
$$\begin{aligned} x+3y-2z&=4\\ y-z&=0\\ z&=-5 \end{aligned}$$

Notice that it is already in row echelon form. Thus, we only need to back substitute to solve. We get

$$\begin{aligned} y-(-5)&=0\\ y+5&=0\\ y&=-5 \end{aligned} \qquad \begin{aligned} x-3(-5)-2(-5)&=4\\ x+15+10&=4\\ x&=-21 \end{aligned}$$

So the solution is $(-21, -5, -5)$.

In a similar way to systems of two variables, we have the possibility of having one solution, no solution or infinite solutions. However, this time we are dealing with one more variable. So geometrically, these types of equations are planes in three dimensions. So here we are talking about the different ways two planes can intersect. The following shows different ways that three planes can intersect.



One solution　　Infinite solutions　　No solutions

Just like in the last section, it is usually fairly clear when we have one of these situations. We either end up with a nonsense statement (like 0 = 3, for example) which means no solution, or we end up with a statement that is always true (like 0 = 0).

Example 3:

Solve by Gaussian Elimination.

a. $\begin{aligned} 2x+y-z&=4\\ y+3z&=2\\ 3x+2y&=4 \end{aligned}$　　b. $\begin{aligned} 2x+z&=1\\ 5y-3z&=2\\ 6x+20y-9z&=11 \end{aligned}$

Solution:

a. Again we will solve by getting the system into row echelon form. Notice that we already have somewhat of a stair step pattern. So we will start by doing ½E_1. We get the system

$$\begin{aligned} x+\tfrac{1}{2}y-\tfrac{1}{2}z&=2\\ y+3z&=2\\ 3x+2y\quad&=4 \end{aligned}$$

Now we will do -3E_1 + E_3 to replace E_3. We get

$$\begin{aligned} -3\left(x+\tfrac{1}{2}y-\tfrac{1}{2}z\right)&=-3(2)\\ 3x+2y\quad&=4 \end{aligned} \Rightarrow \begin{aligned} -3x-\tfrac{3}{2}y+\tfrac{3}{2}z&=-6\\ \underline{3x+2y\qquad}&\underline{=4}\\ \tfrac{1}{2}y+\tfrac{3}{2}z&=-2 \end{aligned}$$

So our system is

$$\begin{aligned} x+\tfrac{1}{2}y-\tfrac{1}{2}z&=2\\ y+3z&=2\\ \tfrac{1}{2}y+\tfrac{3}{2}z&=-2 \end{aligned}$$

Lets multiply E_3 by 2. We get

$$\begin{aligned} x+\tfrac{1}{2}y-\tfrac{1}{2}z&=2\\ y+3z&=2\\ y+3z&=-2 \end{aligned}$$

Now we do –E_2 + E_3. We get

$$\begin{aligned} -(y+3z)&=-(2)\\ y+3z&=-2 \end{aligned} \Rightarrow \begin{aligned} -y-3z&=-2\\ \underline{y+3z}&\underline{=-2}\\ 0&=-4 \end{aligned}$$

Whenever we see a statement of this form we know that the system must have no solution.

b. Again, we will start by doing -3E_1 + E_3 to replace E_3. We get

$$\begin{aligned} -3(2x\qquad+z)&=-3(1)\\ 6x+20y-9z&=11 \end{aligned} \Rightarrow \begin{aligned} -6x\qquad-3z&=-3\\ \underline{6x+20y-9z}&\underline{=11}\\ 20y-12z&=8 \end{aligned}$$

So our system is now

$$\begin{aligned} 2x\qquad+z&=1\\ 5y-3z&=2\\ 20y-12z&=8 \end{aligned}$$

Notice that the last two equations are multiples. Thus lets go ahead and perform the operation -4E_2 + E_3. We get

$$\begin{aligned} -4(5y-3z)&=-4(2)\\ 20y-12z&=8 \end{aligned} \Rightarrow \begin{aligned} -20y+12z&=-8\\ \underline{20y-12z}&\underline{=8}\\ 0&=0 \end{aligned}$$

So, again, when we get a statement of this form we know that the system has infinite solutions.

Lastly, we want to see some applications of larger systems of equations. Many of the applications are similar to what we have done earlier in this text. The only difference is we can now set them up using as many variables as needed. This, in turn can prove to streamline the entire problem. Thus the following example is an example of a problem we have not yet encountered.

<u>Example 4</u>:

Find the constants *a*, *b*, and *c* so that the parabola y = $ax^2 + bx + c$ passes through the three points (1, -2), (-1, 0) and (2, 3).

Solution:
First we need to set up a system of equations. Since we know that each of the given points is on the graph, they each must satisfy the equation. Therefore, we can substitute the values of x and y into the equation for each given ordered pair. We get

<u>For (1, -2):</u>	<u>For (-1, 0):</u>	<u>For (2, 3):</u>
$-2 = a(1)^2 + b(1) + c$	$0 = a(-1)^2 + b(-1) + c$	$3 = a(2)^2 + b(2) + c$
$-2 = a + b + c$	$0 = a - b + c$	$3 = 4a + 2b + c$

So this is our system of equations. It has variables *a*, *b*, and *c*.

$$\begin{aligned} a+b+c&=-2\\ a-b+c&=0\\ 4a+2b+c&=3 \end{aligned}$$

We solve the system by Gaussian Elimination as follows.

$$\begin{array}{c} a+b+c=-2\\ a-b+c=0\\ 4a+2b+c=3 \end{array} \overset{-E_1+E_2}{\Rightarrow} \begin{array}{c} a+b+c=-2\\ -2b\quad=2\\ 4a+2b+c=3 \end{array} \overset{-4E_1+E_3}{\Rightarrow} \begin{array}{c} a+b+c=-2\\ -2b\quad=2\\ -2b-3c=11 \end{array} \overset{-E_2+E_3}{\Rightarrow} \begin{array}{c} a+b+c=-2\\ -2b\quad=2\\ -3c=9 \end{array}$$

$$\overset{\frac{-1}{2}E_2 \text{ and } \frac{-1}{3}E_3}{\Rightarrow} \begin{array}{c} a+b+c=-2\\ b\quad=-1\\ c=-3 \end{array}$$

So now we simply back substitute to solve. We get

$$\begin{aligned} a+(-1)+(-3)&=-2\\ a-4&=-2\\ a&=2 \end{aligned}$$

So, $a=2$, $b=-1$, and $c=-3$.

13.2 Exercises

Solve by Gaussian Elimination.

1. $\begin{aligned} x-2y+4z&=4\\ y-\tfrac{1}{3}z&=\tfrac{2}{3}\\ z&=-5 \end{aligned}$

2. $\begin{aligned} x-2y+4z&=4\\ y&=3\\ z&=-1 \end{aligned}$

3. $\begin{aligned} x+\tfrac{4}{5}y-\tfrac{1}{5}z&=0\\ y-\tfrac{3}{10}z&=\tfrac{11}{3}\\ z&=-5 \end{aligned}$

4. $\begin{aligned} x+y+2z&=9\\ y-\tfrac{7}{2}z&=-\tfrac{17}{2}\\ z&=3 \end{aligned}$

5. $\begin{aligned} x&=10\\ 3x+2y&=2\\ x+y+2z&=0 \end{aligned}$

6. $\begin{aligned} x+z&=4\\ y&=2\\ 4x+z&=7 \end{aligned}$

7. $$\begin{aligned} x \qquad &= 3 \\ -x + y \qquad &= 3 \\ y + 2z &= 4 \end{aligned}$$

8. $$\begin{aligned} 2y + z &= 7 \\ 2x \qquad - z &= 3 \\ x - y \qquad &= 3 \end{aligned}$$

9. $$\begin{aligned} 3x + y \qquad &= 5 \\ 3y - z &= 2 \\ x \qquad + z &= 5 \end{aligned}$$

10. $$\begin{aligned} 2x \qquad + 3z &= 5 \\ 3y + 2z &= 3 \\ 3x + 4y \qquad &= -10 \end{aligned}$$

11. $$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ 3x \qquad - z &= 0 \end{aligned}$$

12. $$\begin{aligned} x + y + z &= 2 \\ -x + 3y + 2z &= 8 \\ 4x + y \qquad &= 4 \end{aligned}$$

13. $$\begin{aligned} x + y + 8z &= 3 \\ 2x + y + 11z &= 4 \\ x \qquad + 3z &= 0 \end{aligned}$$

14. $$\begin{aligned} 2x + y - z &= 4 \\ y + 3z &= 2 \\ 3x + 2y \qquad &= 4 \end{aligned}$$

15. $$\begin{aligned} 3x + y + z &= 2 \\ 4x \qquad + 2z &= 1 \\ 5x - y + 3z &= 0 \end{aligned}$$

16. $$\begin{aligned} 2x \qquad + z &= 1 \\ 5y - 3z &= 2 \\ 6x + 20y - 9z &= 11 \end{aligned}$$

17. $$\begin{aligned} x - y + 2z &= -4 \\ 3x + y - 4z &= -6 \\ 2x + 3y - 4z &= 4 \end{aligned}$$

18. $$\begin{aligned} x + y + z &= -3 \\ 4x + y - 3z &= 11 \\ 2x - 3y + 2z &= 9 \end{aligned}$$

19. $$\begin{aligned} x + 2y + 6z &= 5 \\ -x + y - 2z &= 3 \\ x - 4y - 2z &= 1 \end{aligned}$$

20. $$\begin{aligned} x + 2y - 2z &= 4 \\ 2x + 5y - 7z &= 5 \\ 3x + 7y - 9z &= 10 \end{aligned}$$

21. $$\begin{aligned} 2x + y + 3z &= 1 \\ 2x + 6y + 8z &= 3 \\ 6x + 8y + 18z &= 5 \end{aligned}$$

22. $$\begin{aligned} 4x + y - 3z &= 11 \\ 2x - 3y + 2z &= 9 \\ x + y + z &= -3 \end{aligned}$$

23. $$\begin{aligned} 2x - 5y + z &= 10 \\ -x + 3y \qquad &= -4 \\ x - 2y + 2z &= 9 \end{aligned}$$

24. $$\begin{aligned} 2x + 13y + 6z &= 1 \\ x \qquad + 3z &= 7 \\ 2x + 10y + 8z &= 8 \end{aligned}$$

25. $$\begin{aligned} 3x - 2y + 3z &= -4 \\ 2x + y - 3z &= 2 \\ 3x + 4y + 5z &= 8 \end{aligned}$$

26. $$\begin{aligned} 3x - 3y + 4z &= 6 \\ 4x - 5y + 2z &= 10 \\ x - 2y + 3z &= 4 \end{aligned}$$

27. $$\begin{aligned} 3x - y + 2z &= 2 \\ 4x + 2y - 7z &= 0 \\ 2x + 3y - 5z &= 7 \end{aligned}$$

28. $$\begin{aligned} 2x + 2y + 3z &= 13 \\ -3x + 4y - z &= 5 \\ 5x - 3y + z &= 2 \end{aligned}$$

29. $$\begin{aligned} 2x - 3y + 7z &= 0 \\ x + 4y - 4z &= -2 \\ 3x + 2y + 5z &= 1 \end{aligned}$$

30. $$\begin{aligned} 5x + 3y - z &= 5 \\ 3x - 2y + 4z &= 13 \\ 4x + 3y + 5z &= 22 \end{aligned}$$

31. $$\begin{aligned} 2x + y - 3z &= -4 \\ 4x - 2y + z &= 9 \\ 3x + 5y - 2z &= 5 \end{aligned}$$

32. $$\begin{aligned} 4x - y - z &= 4 \\ 2x + y + z &= -1 \\ 6x - 3y - 2z &= 3 \end{aligned}$$

33. $$\begin{aligned} x + y + z &= 182 \\ y &= 2 + 3x \\ z &= 80 + x \end{aligned}$$

34. $$\begin{aligned} x + y + z &= 57 \\ y &= 2x + 3 \\ x &= z + 6 \end{aligned}$$

35. $$\begin{aligned} 2x - 2y + 2z &= 8 \\ 5x + 2y - 3z &= 2 \\ 4x + 3y - 4z &= -2 \end{aligned}$$

36. $$\begin{aligned} -2x + 8y + 2z &= 4 \\ x + 6y + 3z &= 4 \\ 3x - 2y + z &= 0 \end{aligned}$$

37. $$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$$

38. $$\begin{aligned} y + 3z - 2w &= 0 \\ 2x + y - 4z + 3w &= 0 \\ 2x + 3y + 2z - w &= 0 \\ -4x - 3y + 5z - 4w &= 0 \end{aligned}$$

39. $$\begin{aligned} 2x + 2y + 4z &= 0 \\ -y - 3z + w &= 0 \\ 3x + y + z + 2w &= 0 \\ x + 3y - 2z - 2w &= 0 \end{aligned}$$

40. $$\begin{aligned} 2x - y + 3z + 4w &= 9 \\ x \qquad - 2z + 7w &= 11 \\ 3x - 3y + z + 5w &= 8 \\ 2x + y + 4z + 4w &= 10 \end{aligned}$$

Find the constants a, b, and c so that the parabola $y = ax^2 + bx + c$ passes through the given points.

41. (1, 0), (2, -1), (3, 0)
42. (1, 2), (2, 1), (3, -4)
43. (-1, -3), (1, 1), (2, 0)
44. (-1, -1), (1, 1), (2, -4)

Find the constants D, E, and F so that the circle $x^2 + y^2 + Dx + Ey + F = 0$ passes through the given points.

45. (0, 0), (0, 2), (3, 0)
46. (3, -1), (-2, 4), (6, 8)
47. (-3, 5), (4, 6), (5, 5)
48. (5, 13), (17, 5), (10, 12)

Find the position equation $s = \frac{1}{2}at^2 + v_0t + s_0$ for an object that has the indicated values.

49. s = 128 feet at t = 1 second
s = 80 feet at t = 2 seconds
s = 0 feet at t = 3 seconds

50. s = 48 feet at t = 1 second
s = 64 feet at t = 2 seconds
s = 48 feet at t = 3 seconds

51. s = 32 feet at t = 1 second
s = 32 feet at t = 2 seconds
s = 0 feet at t = 3 seconds

52. s = 10 feet at t = 1 second
s = 54 feet at t = 2 seconds
s = 46 feet at t = 3 seconds

53. A recent basic model of the XRT sport coupe had a price of $12,685. With automatic transmission and power windows the price was $14,070. With air conditioning and power windows the price was $13,580. With air conditioning and automatic transmission the price was $13,925. What was the price of air conditioning, automatic transmission, and power windows individually?

54. Frank Smith has three rock polishers. When they are all working, Frank can polish 570 rocks in a week. When only the first two are working, Frank can polish only 340 rocks in a week. When only the last two are working, Frank can only polish 420 rocks in a week. How many rocks can each machine polish in a week?

55. John, Andrew and Ethan work for company that makes Christmas ornaments. Working together, they can make 74 ornaments a day. On the days that Ethan is off, they can make 44 ornaments a day. On the days Andrew is off, they can make 50 ornaments a day. How many can each person make alone in a day?

56. Emma, Jennifer and Liz volunteer for a local church stuffing envelopes. In one day, together they can stuff 740 envelopes. Emma and Jennifer together can stuff 470 envelopes in a day. Jennifer and Liz together can stuff 520 envelopes in a day. How many envelopes can each person stuff in a day working alone?

57. In a recent basketball game, the College of the Sequoias basketball team scored a total of 90 points. This consisted of 3-pointers, 2 pointers and foul shots worth one point. All together the team made 48 baskets and 24 more 2 pointers than 3 pointers. How many of each kind of shot was made?

58. A standard par 72 golf course is made up of three types of holes: par 3 holes, par 4 holes and par 5 holes. There is the same number of par 3 holes as par 5 holes, and there are two more par 4 holes than the total of all the par 3 and par 5 holes. How many of each type of hole is on a standard 18 hole golf course?

59. Matt has a bunch of change in the ash tray of his car. Since Matt hates pennies, he only has nickels, dimes and quarters in the ash tray. The total value of the cash in his ash tray is $1.85. There are twice as many dimes as quarters and there 19 coins in all. How many of each time of coin does Matt have in his ash tray?

60. The change from a coin operated washing machine is collected once a week. The machine only accepts nickels, dimes and quarters. One particular week the machine had 430 coins totaling $69.85. There were 18 less nickels than twice the number of dimes. How many of each type of coin was in the machine?

61. In the card game Cribbage there are three types of 3 card runs: a run of 3- worth 3 points, a double run of 3- worth 8 points, and a double-double run of 3- worth 16 points. If a player had 8 total 3 card runs which gave him 52 points and had 2 more double runs than double-double runs, how many of each type of 3 card runs did the player have?

62. In the card game Cribbage you can score points for a pair, three of a kind and four of a kind. A pair is worth 2 points, a three of a kind is worth 6 points and a four of a kind is worth 12 points. If a player had 11 total pairs, three of a kinds and four of a kinds, that gave her 76 points, how many pairs did the player have?