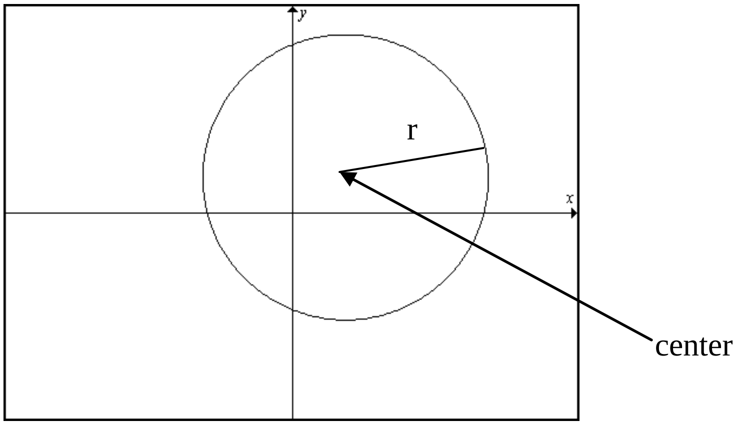
12.3 The Circle

The next conic that we would like to examine is the circle.

Standard form of a Circle
Let r be the radius of a circle and let (h, k) be the center of the circle. Then the equation of the circle is $(x-h)^2 + (y-k)^2 = r^2$

This gives us the following picture



Graphing a circle is fairly easy. All we need to do is get the equation into standard form and then plot the center and use the radius to determine the rest. We will also want to find the x-and y-intercepts.

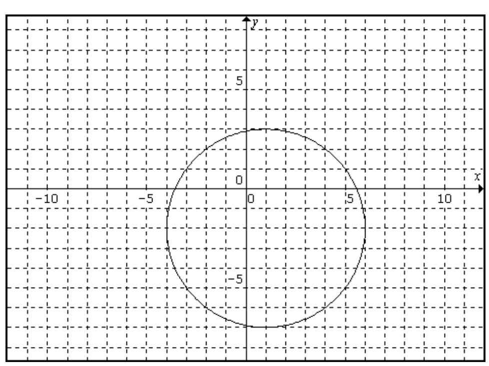
<u>Example 1</u>:

Graph the following. Find all x and y-intercepts.

a. $(x-1)^2 + (y+2)^2 = 25$ b. $x^2 + y^2 + 8x - 2y + 13 = 0$

Solution:

a. Since the equation is already in standard form, we can simply read off the center of the circle. Notice that the equation is $(x-h)^2 + (y-k)^2 = r^2$ and the center is (h, k). That means the center is $(1, -2)$ in our case. We can also read off the radius. Since we see that $r^2 = 25$ we clearly have $r = 5$. Now to graph, we simply plot the center and go out from it 5 units in each direction. We get the following graph



Now we still need to find the x- and y-intercepts. We do this the same way we always have. For x-intercepts, set y = 0 and for y-intercepts, set x = 0.

x-intercepts:

$$\begin{aligned}(x-1)^2+(0+2)^2&=25\\(x-1)^2+4&=25\\(x-1)^2&=21\\x&=1\pm\sqrt{21}\\x&\approx 5.6,\ -3.6\end{aligned}$$

y-intercepts:

$$\begin{aligned}(0-1)^2+(y+2)^2&=25\\1+(y+2)^2&=25\\(y+2)^2&=24\\y&=-2\pm 2\sqrt{6}\\y&\approx -6.9,\ 2.9\end{aligned}$$

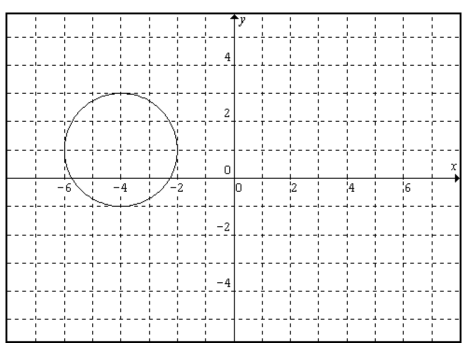
Looking at the graph again we can see clearly these are the correct values.

b. This time we are not given the equation in such a nice form. We will need to put the equation into standard form in order to get the important information. To do that, we will complete the square just like in the previous section. However, this time, since we want to have the radius on the right hand side, we will add our completed square portion to both sides instead of adding it in and then subtracting it off of the same side as we did with the parabola. We proceed as follows

$$\begin{aligned}x^2+y^2+8x-2y+13&=0\\x^2+8x\quad+y^2-2y\quad&=-13\\x^2+8x+\left(\tfrac{1}{2}\cdot 8\right)^2+y^2-2y+\left(\tfrac{1}{2}\cdot 2\right)^2&=-13+\left(\tfrac{1}{2}\cdot 8\right)^2+\left(\tfrac{1}{2}\cdot 2\right)^2\\(x+4)^2+(y-1)^2&=-13+16+1\\(x+4)^2+(y-1)^2&=4\end{aligned}$$

Now we can clearly see that the center is $(-4,1)$ and the radius is $r=2$.

So we graph the circle by plotting the center and going out 2 units in each direction. We get



Lastly we need to find the x- and y-intercepts.

We can see from the graph that we should have two x-intercepts and no y-intercepts since it hits the x-axis twice and does not hit the y-axis at all. Lets verify this with usual steps.

x-intercepts:

$$(x+4)^2+(0-1)^2=4$$
$$(x+4)^2+1=4$$
$$(x+4)^2=3$$
$$x=-4\pm\sqrt{3}$$
$$x\approx-5.7,\,-2.3$$

y-intercepts:

$$(0+4)^2+(y-1)^2=4$$
$$16+(y-1)^2=4$$
$$(y-1)^2=-12$$
$$y=1\pm 2i\sqrt{3}$$

So we can see that since we get imaginary values for our y-intercepts, we have no y-intercepts, just as our graph had indicated.

One of the main things that we want to do in this chapter is to be able to distinguish between the various conics from the very beginning of the problem. So far the only conics that we have are the parabola and circle. It is very easy to tell a parabola from all the other conics, since the parabola will be the only one that only has one variable that is squared.

So, at the moment we have the following
Only one of the two variables is squared- the conic is a parabola
Both variables are squared- the conic is a circle

12.3 Exercises

Graph the following. Identify the center, radius and all x and y-intercepts.

1. $x^2+y^2=4$
2. $x^2+y^2=9$
3. $(x-1)^2+y^2=9$
4. $x^2+(y-2)^2=4$
5. $(x-2)^2+(y-3)^2=16$
6. $(x-1)^2+(y-2)^2=25$
7. $(x+2)^2+(y-1)^2=25$
8. $(x-4)^2+(y+3)^2=16$
9. $(x+7)^2+(y+4)^2=81$
10. $(x+5)^2+(y-4)^2=49$
11. $x^2+y^2+4x+2y-4=0$
12. $x^2+y^2+6x+4y-3=0$
13. $x^2+y^2-8x-4y-29=0$
14. $x^2+y^2-2x+4y-31=0$
15. $x^2+y^2+10x-6y-2=0$
16. $x^2+y^2+2x-4y+4=0$
17. $x^2+y^2-2x-2y+1=0$
18. $x^2+y^2+8x-2y+8=0$
19. $x^2+y^2-4x+10y+20=0$
20. $x^2+y^2-12x+8y+48=0$
21. $x^2+y^2-10x-10y+34=0$
22. $x^2+y^2-10x=0$
23. $x^2+y^2+6y+7=0$
24. $x^2+y^2+2x-2y-10=0$
25. $x^2+y^2-6x+4y+5=0$
26. $x^2+y^2-6x-8y+20=0$

Identify if the following is a parabola or a circle. Then graph accordingly.

27. $y^2+y+x-2=0$
28. $x^2+y^2+8x-6y+21=0$
29. $x^2+y^2+4x-2y-11=0$
30. $-x^2+2x+y+4=0$
31. $x^2+y^2-4x-10y+28=0$
32. $x^2+y^2+4x-2y+4=0$
33. $2x^2+4x-y=0$
34. $3y^2-9y+x-7=0$

Find the equation of the circle that satisfies the given conditions.

35. center $(-1,\ 3)$, $r = 3$

36. center $(2,\ -5)$, $r = 4$

37. center $(-1,\ 1)$ and point $(3,\ 4)$ is on the circle

38. center $(-2,\ -3)$ and point $(1,\ 1)$ is on the circle

39. $(-1,\ 1)$ and $(3,\ 4)$ are endpoints of the diameter

40. $(-2,\ -3)$ and $(1,\ 1)$ are endpoints of the diameter

41. center $(-2,\ 1)$ and one y-intercept is at –3

42. center $(4,\ -5)$ and one x-intercept is at 3

Find the area of the following circles.

43. $(x-1)^2 + (y-2)^2 = 25$

44. $(x+2)^2 + (y-1)^2 = 16$

45. $x^2 + y^2 + 6x + 4y - 3 = 0$

46. $x^2 + y^2 - 8x - 4y - 29 = 0$

47. $x^2 + y^2 - 2x + 4y - 31 = 0$

48. $x^2 + y^2 + 10x - 6y - 2 = 0$

49. $x^2 + y^2 + 2x - 4y + 4 = 0$

50. $x^2 + y^2 - 6x + 4y + 5 = 0$

51. $x^2 + y^2 - 6x - 8y + 20 = 0$