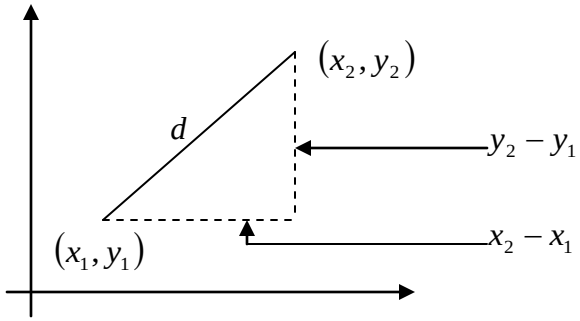
12.1 The Distance and Midpoint formulas

In this section we want to take a look at two very useful formulas: the distance and midpoints formulas.

First,

The Distance Formula
The distance *d* between any two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

This formula is very easy to derive. It comes from the Pythagorean theorem as follows:



So by the Pythagorean theorem we have $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$d = \pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

But since we are talking about the distance (or length) we cannot have a negative. Thus,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1:

Find the distance between the following points:

(5, 21) and (-3, 1) & (5.9, 2) and (3.7, -7.7)

Solution:
We simply need to label our points and insert them into the distance formula.
Lets label (5, 21) as (x_1, y_1) and (-3, 1) as (x_2, y_2). Inserting them into the formula we get

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-5)^2 + (1-21)^2} \\ &= \sqrt{(-8)^2 + (-20)^2} \\ &= \sqrt{64 + 400} \\ &= \sqrt{464} \\ &= 4\sqrt{29} \approx 21.54 \end{aligned}$$

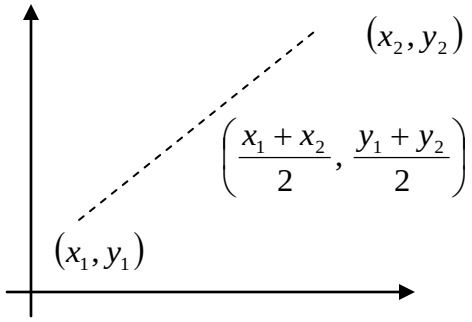
Similarly we label (5.9, 2) as (x_1, y_1) and (3.7, -7.7) as (x_2, y_2). We get

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(3.7 - 5.9)^2 + (-7.7 - 2)^2} \\
&= \sqrt{(-2.2)^2 + (-9.7)^2} \\
&= \sqrt{4.84 + 94.09} \\
&= \sqrt{98.93} \\
&\approx 9.95
\end{aligned}
$$

The other formula we wanted to discuss in this section is the midpoint formula. The midpoint is defined as the point that is exactly half way between two given points. The formula is as follows:

The Midpoint Formula
The midpoint of (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

This formula is also easy to see. The point halfway between x_1 and x_2 is $\frac{x_1 + x_2}{2}$ and the point halfway between y_1 and y_2 is $\frac{y_1 + y_2}{2}$. We get the following picture



Example 2:

Find the midpoint of the line segment connecting the following points:

(-1, 2) and (1, -3) & $(-\frac{4}{5}, -\frac{2}{3})$ and $(\frac{1}{8}, \frac{3}{4})$

Solution:
In a similar fashion to the distance formula we simply need to label the points and insert them into the corresponding formula.

Lets label (-1, 2) as (x_1, y_1) and (1, -3) as (x_2, y_2). Inserting them into the formula we get

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+1}{2}, \frac{2+(-3)}{2}\right)$$
$$= \left(\frac{0}{2}, \frac{-1}{2}\right)$$
$$= \left(0, -\frac{1}{2}\right)$$

Similarly we label $(-\frac{4}{5}, -\frac{2}{3})$ as (x_1, y_1) and $(\frac{1}{8}, \frac{3}{4})$ as (x_2, y_2). We get

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-\frac{4}{5}+\frac{1}{8}}{2}, \frac{-\frac{2}{3}+\frac{3}{4}}{2}\right)$$
$$= \left(\frac{-\frac{27}{40}}{2}, \frac{\frac{1}{12}}{2}\right)$$
$$= \left(-\frac{27}{80}, \frac{1}{24}\right)$$

12.1 Exercises

Find the distance between the given points.

1. $(0, 0), (3, -4)$
2. $(2, 4), (5, 8)$
3. $(-2, -8), (3, 4)$
4. $(5, 3), (7, 9)$
5. $(5, 6), (7, 10)$
6. $(2, -7), (-3, 12)$
7. $(-5, -2), (7, 3)$
8. $(10, 4), (2, -2)$
9. $(6, 5), (4, 2)$
10. $(-8, 12), (3, -9)$
11. $(4, 6), (10, 6)$
12. $(1, 2), (5, 6)$
13. $(7, 3), (5, 1)$
14. $(3, 5), (1, -3)$
15. $(4, -2), (-3, -6)$
16. $(6, -5), (-2, -1)$
17. $(\frac{1}{2}, \frac{1}{4}), (\frac{1}{3}, \frac{1}{2})$
18. $(-\frac{4}{5}, -\frac{2}{3}), (\frac{1}{8}, \frac{3}{4})$
19. $(0, \sqrt{7}), (\sqrt{6}, 0)$
20. $(\sqrt{8}, \sqrt{3}), (-\sqrt{5}, -\sqrt{6})$
21. $(9, 2\sqrt{3}), (-4, 5\sqrt{3})$
22. $(x, y), (a, b)$
23. $(2, 3a), (-1, a)$
24. $(p, q), (q, p)$

Find the midpoint of the line segment connecting the points.

25. $(5, 3), (7, 9)$
26. $(5, 6), (7, 10)$
27. $(2, -7), (-3, 12)$
28. $(0, 0), (3, -4)$
29. $(2, 4), (5, 8)$
30. $(-2, -8), (3, 4)$
31. $(-8, 12), (3, -9)$
32. $(4, 6), (10, 6)$
33. $(1, 2), (5, 6)$
34. $(-5, -2), (7, 3)$
35. $(10, 4), (2, -2)$
36. $(6, 5), (4, 2)$
37. $(3, 5), (1, -3)$
38. $(7, 3), (5, 1)$
39. $(6, -5), (-2, -1)$
40. $(4, -2), (-3, -6)$
41. $(-\frac{4}{5}, -\frac{2}{3}), (\frac{1}{8}, \frac{3}{4})$
42. $(\frac{1}{2}, \frac{1}{4}), (\frac{1}{3}, \frac{1}{2})$
43. $(0, \sqrt{7}), (\sqrt{6}, 0)$
44. $(9, 2\sqrt{3}), (-4, 5\sqrt{3})$
45. $(\sqrt{8}, \sqrt{3}), (-\sqrt{5}, -\sqrt{6})$
46. $(x, y), (a, b)$
47. $(p, q), (q, p)$
48. $(2, 3a), (-1, a)$