11.3 Graphing Logarithmic Functions

Next we want to look at the graphs of logarithmic functions.

First we recall that $f(x) = a^x$ and $f(x) = \log_a x$ are inverse functions by construction. Therefore, we can graph $f(x) = \log_a x$ by using all of our knowledge about inverse functions and the graph of $f(x) = a^x$.

Recall the following facts from inverse functions: If the point (a, b) is on the graph of *f* then (b, a) is on the graph of f^{-1}. This corresponds to a reflection about the line y = x. And the domain and range of a function are the range and domain of the inverse, respectively.

<u>Example 1</u>:

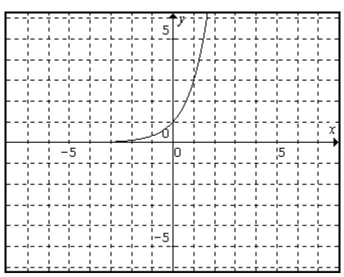
Graph $f(x) = 3^x$ and $f(x) = \log_3 x$.

Solution:

First we will graph $f(x) = 3^x$. Lets plot a few points like we did in section 11.1.

x	f(x)
-1	1/3
0	1
1	3

This gives us

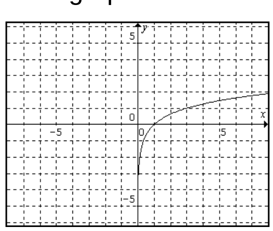


So now using the facts about inverse functions we can take all the points in our table and reverse the order for $f(x) = \log_3 x$. This should give us a reflection across the line y = x. Our table of values is

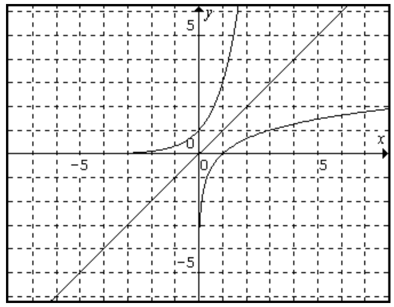
x	f(x)
1/3	-1
1	0
3	1

Also, since the graph of $f(x) = 3^x$ never crosses the x- axis, the graph of $f(x) = \log_3 x$ never crosses the y-axis.

Putting this together we get the graph



If we graph both functions together we can clearly see the reflection across the line y = x.



Summary of the graph of y = log$_a$ x, for a > 0, a$\neq$1
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: (1, 0)

We get the domain and range both graphically and from the fact that the inverse has the domain and range of the function switched.

So consequently, we see that the domain in composed of only positive numbers. This means we can never have a 0 or negative inside of a logarithm. We saw this fact in the last section.

We can also see, since when we graphed exponential functions we had to plot a few points because the size of the base changed (slightly) the shape of the graph, we will have to plot a few points for logarithms as well. However, since the domain in limited to only positive values, we have to be careful how we do that. We will see exactly how to choose your values in the next example.

The last thing we need to notice about logs is that since they are related to the exponential function and all of our tricks for shifting, reflecting and dilating worked for exponential functions, it seems to reason that they would work on logarithmic functions as well. As it turns out this is the case.

<u>Example 2</u>:

Graph h(x) = $\log_2$ (x + 1).

Solution:

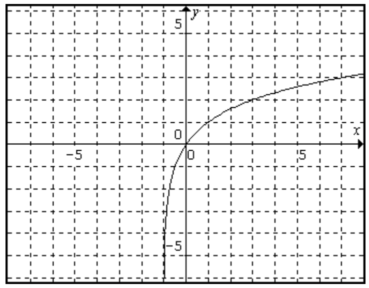
First, like before, lets identify the basic function involved. We can see that is y = $\log_2$ x. Clearly, we have a shift of 1 left since we have a one added on the inside of the log.

Now, since the shape of the graph is different for each base we must plot a few points to account for that difference. Looking at the table of values from example 1 we can see the values that were use were all powers of the base. Thus, that what we should use for our x values here: Powers of the base, which is 2. If we do that, the log will always return to us what power it is. So we will use ½, 1 and 2; the –1, 0 and 1st powers of 2.
We get

x	y=$\log_2$x
½	-1
1	0
2	1

Now we simply need to plot the basic graph and shift it accordingly.

We get a final graph of



<u>Example 3</u>:

Graph $f(x) = -\log_7 (x - 2) +3$.

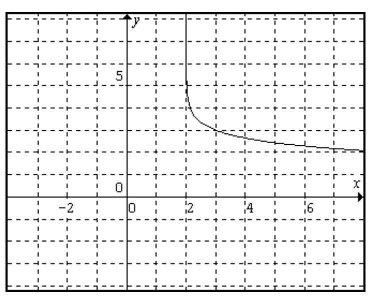
Solution:

Again the basic function is $y = \log_7 x$. This time we have a reflection across the x-axis, a shift 2 right and 3 up.

So we start by plotting a few points on the basic function. Again we want to use powers of the base for our x values to simplify our points. Since the base is 7 we use 1/7, 1 and 7.
We get

x	$y=\log_7 x$
1/7	-1
1	0
7	1

We plot the basic graph and remember, we like to do the reflections first and then the shifting.
We get a final graph of



11.3 Exercises

Graph the given exponential function and use it to graph the logarithmic function.

1. $y = 2^x$, $y = \log_2 x$
2. $y = 4^x$, $y = \log_4 x$
3. $y = 5^x$, $y = \log_5 x$
4. $y = 6^x$, $y = \log_6 x$
5. $y = e^x$, $y = \ln x$
6. $y = ½^{\,x}$, $y = \log_{1/2} x$

Graph the following.

7. $f(x) = \log_2 x + 1$
8. $g(x) = \log_3 x – 2$
9. $h(x) = \log_3 (x-1)$
10. $f(x) = \log_4 (x + 2)$
11. $g(x) = 4 + \log_3 x$
12. $h(x) = -\log_5 x$
13. $f(x) = -\log_4 (-x)$
14. $g(x) = -\log_3 x + 2$
15. $h(x) = -\log_2 x – 4$
16. $f(x) = -\log_6 (x - 3)$
17. $g(x) = -\log_7 (x + 4)$
18. $h(x) = \log_2 (-(x + 1))$
19. $f(x) = -\log_5 (-x)$
20. $g(x) = \log_5 (x + 2) - 3$
21. $h(x) = \log_3 (x - 3) + 1$
22. $f(x) = 2 + \log_5 (x + 2)$
23. $g(x) = 3 + \log_3 (x - 3)$
24. $h(x) = 1 – \log_3 (-x)$
25. $f(x) = \log_7 (x - 1) + 2$
26. $g(x) = 2 – \log_2 (x + 1)$
27. $h(x) = 1 – \log_3 (x - 1)$

28. $f(x) = -\log_6 (x + 1) - 1$

29. $g(x) = \log_5 (-(x - 3)) - 1$

30. $h(x) = -\log_2 (x - 1) + 4$

31. $f(x) = -\log_2 (-x) - 1$

32. $g(x) = 2 + \log_4 (x + 2)$

33. $h(x) = 5 – \log_6 (x + 3)$

34. $f(x) = \log_5 (-(x + 4)) - 2$

35. $g(x) = \log(x - 2) - 3$

36. $h(x) = \log(x + 1) + 1$

37. $f(x) = 4 – \log_5 (-(x + 3))$

38. $g(x) = -\log_4 (-(x + 1)) + 1$

39. $h(x) = -4 – \log_3 (-(x - 4))$

40. $f(x) = \log_{1/2} x$

41. $g(x) = \log_{1/3} x$

42. $h(x) = \log_{1/2} (x + 2)$

43. $f(x) = \log_{1/3} (x + 1)$

44. $g(x) = -\log_{1/2} (x + 2)$

45. $h(x) = \log_{1/3} (-(x + 1))$

46. $f(x) = 4 – \log_{1/2} (x + 2)$

47. $g(x) = \log_{1/3} (-(x + 1)) + 3$

48. $h(x) = \ln x$

49. $f(x) = \ln x + 1$

50. $g(x) = \ln (x - 1)$

51. $h(x) = \ln (x + 2) - 3$